Reputation and Competition Among Information Intermediaries

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Abstract

This paper investigates the effect of competition on the reputation mechanism in the market for credit rating agenies. I use a dynamic model to endogenize the value of reputation to enable comparison of equilibria under different market structures. Behavior is determined in the model by weighing the current rating fee against the future value the rating agency derives from enjoying a good reputation. I show that competition weakens the reputation mechanism, and thus worsens the quality of information by reducing the value of a good reputation but not the short-term gain of lying.

Information intermediaries can potentially alleviate the problem of asymmetric information. Credit rating agencies convey information about creditworthiness of an issuer of financial product; certification bodies such as standard-setting organizations convey information about whether a product or production process satisfies certain properties. The value of these agencies relies on their ability to credibly convey information about the product they rate.

An information intermediary has a conflict of interest when it is paid by the sellers whose products it evaluates. This issue in the credit rating industry has received considerable public attention in the wake of the recent financial crisis. A report by Levin and Coburn (2011) documents how analysts are under pressure to adjust models to give clients a desirable rating in order not to lose business. Griffin and Tang (forthcoming) show that a top US rating agency frequently adjusts, often upward, the rating its own model predicts.

Such conflict of interest can be mitigated by concern for reputation. If an intermediary delivers poor information, then its ratings will not be highly valued by market participants in the future, and thus it will be unable to charge a high

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fee for its services. S&P explicitly mentions its ongoing value of credit ratings business as cost to counter short-run incentives to inflate ratings.[1]

There seems to be widely-held belief that competition improves the quality of the information that is delivered in the credit rating industry. One goal of the Credit Rating Agency Reform Act of 2006 was to increase competition in this industry by making the requirements for becoming a Nationally Recognized Statistical Ratings Organizations (NRSRO) more explicit. Richard A. Posner proposes eliminating the status of NRSRO altogether to increase competition because market discipline should mitigate conflict of interest.[2] However, an empirical study by Becker and Milbourn (2011) suggest that increased competition in the rating industry results in more issuer-friendly ratings and degrades the quality of information. Levin and Coburn (2011) document incidences suggesting that concern over market share and the threat of losing deals to a competitor may compromise ratings quality.

This paper investigates whether competition strengthens or weakens reputation mechanism among information intermediaries. I employ a fully dynamic model of reputation in which the value of reputation in each period is endogenously determined by the market structure. I build on the monopoly model proposed by Mathis et al.(2009) by adding a fee-setting stage to capture competition. I find that competition reduces market efficiency by causing an agency to lie more often.

As in Mathis et al.(2009), I consider a financial market in which, in every period, a new firm wishes to issue securities to finance a new project. The payoff to this firm depends on investors' beliefs about its project. A rating agency observes the quality of the project and issues a good or bad rating. Some agencies are honest and always report the true quality. Others are opportunistic and provide any report that will maximize its discounted sum of the per period payoff. An agency's reputation is the probability that other market participants believe it to be honest. The firm chooses to obtain a rating from one or no agency after learning the fees each agency charges.

In equilibrium, a project with a bad rating will not be financed, and thus the agency will not be paid because fees are transaction based. An opportunistic agency thus has a short-run incentive to lie in order to receive the rating fee it charges. If it lies, however, it will lose all of its reputation and, as a result, all of its future business. Thus, the cost of lying is the value of a good reputation earned by giving a bad rating. In equilibrium an agency will always lie with positive probability if there are enough bad projects or if the agency is not patient enough. The probability with which it lies is such that the agency is indifferent between lying and not lying. It is the probability at which the two curves in Figure 1 intersect.

Under monopoly, the agency leaves zero surplus to any firm, and charges a fee equal to the value of its good rating. Thus, the benefits of lying are simply the

[1] It is in a statement S&P made in 2002 to SEC. See http://www.sec.gov/news/extra/credrate/standardpoors.htm>

[2] See http://www.finreg21.com/lombard-street/the-president's-blueprint-reforming-financial-regulation-a-critique-part-ii

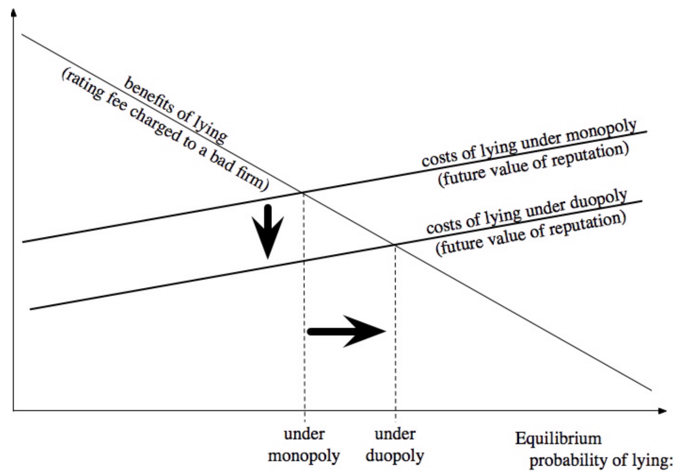


Figure 1: Equilibrium Probability of Lying

value of the monopolist's good rating. The bulk of an agency's profits come from rating good firms. Firms with bad projects are actually hot potatoes. Although an agency gets paid a rating fee when it lies for a bad firm, its future value of business suffers as a result of doing so. Thus, the surplus from trade with a bad firm is nil as long as the agency's reputation is not close to perfect. Therefore, competition cannot further drive down the surplus to reduce the temptation of lying. Competition does, however, erode the more profitable business of rating good firms, thereby reducing the value of having a good reputation, and thus the costs of lying. Because competition does not change the benefits-of-lying curve, but lowers the cost-of-lying curve, competition causes an agency to lie more often, and thus reduces the value of its good ratings (see figure 1).

0.1 Literature Review

Bolton et al.(forthcoming) adopt a static model with an exogenous value of reputation and a fraction of naive investors to examine the phenomenon of ratings shopping. They extend the model to a two-period model to endogenize the value of reputation in the first period, and show that market efficiency is lower under duopoly because it facilitates ratings shopping. The two-period model provides tractability for analyzing the comparative statics of a host of other interesting issues. In contrast, this paper is able to build a fully dynamic model in which the value of reputation is endogenous in every period by focusing solely on the reputation mechanism and abstracting away from ratings shopping. I assume that all investors are rational and correctly interpret a rating given their expectation of a rating agency's equilibrium behavior. In the model employed by Bolton et al.(forthcoming), competition does not make an agency lie more or less often. It is the issuer's better ability to pick the best rating and fool naive investors that make a duopoly perform worse. In my model, competition worsens efficiency because agencies lie more often under duopoly than under monopoly. Therefore, my paper is complementary to theirs by pointing out a different channel through which competition worsens efficiency.

Camanho et al.(2010) also extend Mathis et al.(2009). In contrast to this paper, they assume that rating fee is exogenous and does not depend on market structure. They use numerical examples to argue that duopoly tend to increase ratings inflation.

The literature on the effect of competition on the reputation mechanism typically focuses on the existence of equilibria in which the strategic long run player always takes the good action and the incentives to take the bad action are independent of market structure. Klein & Leffler (1981), Strausz (2005), and McLennan and Park (2005) all show that reputation mechanism acts as entry barrier, which is why the industry is a natural monopoly or can only accommodate a handful of firms. In their models, adding another agency either has no effect on efficiency, or makes the entire industry collapse. Horner (2002), on the other hand, shows that competition among sellers in a product market enhances reputation concern and improves product quality because it improves consumers' outside options and therefore heightens the threat of losing reputation.

Another line of research concerning certification agencies studies the optimal rating rule, assuming that the agency can commit to a disclosure rule and a rating-contingent fee schedule. Lizzeri (1999) shows that the profit-maximizing rating rule for a monopolist agency is a pass-fail system that reveals only whether a product has positive or negative value to the consumer. But competition may lead to full information revelation. In contrast, considering truthful agencies and renegotiation-proof contracts, Faure-Grimaud et al (2009) show that competition may reduce information disclosure. Their model provides a story why competition may cause agencies to optimally give up a section of the market with lower willingness-to-pay for a rating.

This paper is organized as follows: In Section 1, I set up the model and describe the solution concept. In Section 2, I solve for the equilibrium under monopoly. In Section 3, I analyze the duopoly case and compare efficiency in the two market structures. In Section 4, I discuss the assumptions about the model setup. Section 5 concludes.

1 Model Setup

My model builds on the monopoly model in Mathis et al.(2009) with an additional fee-setting stage to capture competition.

In every period $t = 0, 1, 2, ...$, a short-lived firm wishes to seek funds for an investment project by issuing a security. The project can be good or bad. The firm knows the quality of its project, but potential investors do not. The latter hold a prior belief that the project is good with probability λ. The firm's payoff from the financial market is $w(p)$ if investors believe the project to be good with probability p. I assume that no one would invest in the firm and no issue would take place if the investors' beliefs about the firm were no better than $\underline{p} \geq \lambda$. For $p > \underline{p}$, the firm's financial-market payoff $w(p)$ is strictly increasing and continuous in the investors' belief p. In addition to being affected by the price of debt the firm has to pay, $w(p)$ can also reflect the amount of financing the firm is able to receive.

Rating agencies are long-run players whose payoffs are represented by the discounted sum of stage game payoffs with a discount factor $\delta \in (0,1)$. The set of rating agencies is $I = \{1\}$ under monopoly and $I = \{1,2\}$ under duopoly. The agencies perfectly observe the quality of the new investment project when it arrives and can communicate that quality to the market by issuing a rating. A rating agency is either a behaviroal type or a strategic type. A behavioral type cannot lie, i.e., he can only issue a good rating to a good investment project and a bad rating to a bad project. A strategic type will lie if doing so increases its payoff. At the beginning of each period, all players other than agency i hold the same belief about agency i.

Within a given period, the game proceeds as follows.

1. A firm arrives with a new investment project of quality $v \in \{g, b\}$. Both the firm and the agencies observe v.

2. Bertrand competition in rating fee:

    (a) Rating agencies simultaneously post their rating fee ϕ_i.

    (b) The firm chooses one or no agency.

3. Rating choice: the chosen agency i publishes a rating $m \in \{G_i, B_i\}$. If the firm chooses no agency, then $m = \emptyset$.

4. Payoff realization and belief updates:

    (a) The market updates its belief about the firm to p^m.

    (b) If $w(p^m) > 0$, the firm issues securities and gets a net payoff of $w(p^m) - \phi_i$, and the chosen agency i receives ϕ_i. The market then observes the actual quality of the firm.

    (c) If $w(p^m) = 0$, the firm does not issue securities. Therefore, the firm obtains payoff 0, and the chosen agency i does not receive his rating fee. The market does not observe the actual quality of the firm.

    (d) Given everything the market observes, denoted by o, the market updates its belief about every agency to $\chi^o = (\chi_i^o)_{i \in I}$.

I model conflict of interest by assuming that fees are paid only if securities are issued. In equilibrium, the market correctly believes that a firm with a bad rating has a bad project. Thus, an agency will be paid only if he issues a good rating. This captures the short-run pressure to issue a good rating even when the underlying project is bad. In reality, one can think of rating fee as a short hand for all such financial pressure, which may include revenue from future deals with the same firm, or fees from nonrating services provided to the same firm.

1.1 Strategies, Beliefs and the Solution Concept

The solution concept used in this paper is symmetric Markov Perfect Equilibrium. That is, every player's behavior within a given period depends on the history only through the reputation profile of the agencies. Let $q = (q_i)_{i \in I}$ denote the reputation profile at the beginning of a period. The Markovian property implies that the strategic type agency i's lifetime payoff can be written as $V_i(q)$. The symmetry assumption implies that the lifetime payoff to agency i depends only on his own reputation and his opponent's reputation, but not on his identity. So $V_i(q) = V(q_i, q_{-i})$.

I restrict analysis to *fee-pooling* equilibria where the rating fee an agency charges is independent of his type. Therefore, rating fees have no signaling value. Only rating *choice* does. This also allows easier comparison with Mathis et al since in their paper, the agency does not directly choose his rating fee.

I also restrict analysis to equilibria where the agency with a strictly higher reputation has a strictly higher lifetime payoff, i.e. $V_1(q) > V_2(q)$ if $q_1 > q_2$.

Under the following parameter assumption, Mathis et al shows that in every equilibrium, the behavioral type monopolist agency lies with positive probability at every reputation level.

Assumption $\frac{\lambda\delta}{1-\delta} < 1$.

Let $(q_i)_{i \in I}$ denote the reputation profile at the beginning of the period. After observing the quality v of the project, agency i of type t_i posts a fee $\phi_i^v(q; t_i)$. When the fees the agencies charge are $(\phi_i)_{i \in I}$, a firm with a good (bad) project believes that agency i is honest with probability $\gamma_i^g(\phi_i)$ $(\gamma_i^b(\phi_i))$, and will approach agency i with probability $\alpha_i(\phi; q)$ $(\beta_i(\phi; q))$. An opportunistic agency i chooses to give a good rating to a good (bad) firm with probability $y_i(\phi_i; q)$ $(x_i(\phi_i; q))$. Let $(\alpha_i^*(q), \beta_i^*(q))$ be the firm's behavior after equilibrium fees are posted, and $(x_i^*(q), y_i^*(q))$ agency's i's behavior when a firm approaches it at its equilibrium fee.

I will describe an equilibrium under monopoly that exhibits the same behavior as the unique equilibrium in Mathis et al. I call this the MMR equilibrium. I will then construct a fee-pooling equilibrium under duopoly where the agency with a higher reputation has a higher payoff. Both equilibria share the following behaviors:

1. An agency known to be the strategic type has zero lifetime payoff: $V(0) = V(0, q_{-i}) = 0$ for all $q_{-i} \in [0, 1]$.
2. A firm with a bad rating or no rating will not issue securities because $w(p^\emptyset) = w(p^B) = 0$.

3. An agency gives a good rating to a good firm with probability 1.

4. A good firm obtains a rating in equilibrium with probability 1.

It then remains to pin down the rating fee that may depend on project quality, the behavioral type agency's rating choice when the firm has a bad project, and the equilibrium probability a bad firm obtains a rating.

Suppose the market expect the strategic type to lie with probability x and a bad firm obtains a rating with probability β^*. Then the market believes that a bad firm rated by the agency is given a bad rating with probability $a = 1 - (1 - q)\, x$. Then the market believes that a firm with a good rating from the agency has a good project with probability

$$p^G = \frac{\lambda}{\lambda + (1 - \lambda)\, \beta^* \,(1 - a)}.$$

If the agency gives a bad rating, the market then believes that the agency is the behavioral type with probability

$$\chi^B = \frac{q}{a}.$$

The heart of the matter is that, when the firm has a bad project, what will a strategic type rating agency do? If he lies and issues a good rating, then he gets paid his rating fee in the current period. But then the market will find out about the actual quality of the project and realize that the agency has lied. Since a behavioral type never lies, by lying the agency will reveal himself to be the strategic type. His reputation will thus drop to 0. His future payoff will be $V(0)$. So if he lies, his payoff is

$$\phi + \delta V(0)$$

If he does not lie, i.e. if he gives a bad rating, then he does not receive his rating fee in this period. But his reputation next period will become χ^B which we will derive shortly. So if he does not lie, his payoff is

$$0 + \delta V\left(\chi^B\right)$$

Therefore, the monopolist agency's short run gain from lying is equal to his rating fee ϕ. He will charge the firm its willingness-to-pay for a good rating, which is equal to $w\left(p^G\right)$ because obtaing no rating gives the firm $w\left(p^{\emptyset}\right) = 0$. So the monopolist rating agency's short run gain from lying is

$$\begin{aligned} & w\left(p^G\right) \\ = \quad & w\left(\frac{\lambda}{\lambda + (1 - \lambda)\, \beta^* \,(1 - a)}\right) \end{aligned}$$

It is increasing in the market's expected accuracy. Suppose the market believes that, in equilibrium, a strategic type lies with probability x, and a bad firm obtains a rating with probability β^*, then the market believes that a bad firm rated by the agency is given a bad rating with probability $a = 1 - (1-q)\,x$. Therefore, after giving a bad rating, the market believes that the agency is behavioral type with probability

$$\chi^B = \frac{q}{1-(1-q)\,x} = \frac{q}{a}.$$

The cost of lying is the loss from the discounted lifetime payoff from the next period onwards, which is equal to

$$\begin{aligned} & \delta V\left(\chi^B\right) \\ = \; & \delta V\left(\frac{q}{a}\right) \end{aligned}$$

I will construct an equilibrium in which a bad firm obtains a rating with probability 1 in equilibrium at every reputation level $q \in (0,1]$. I call this the MMR equilibrium because the behaviors are exactly the same. It remains to pin down the equilibrium probability of lying at every reputation level: $x^*(q)$. I will then show that the posited behaviors are optimal.

First, given the parameter assumptions, at every reputation level, the strategic type lies with positive probability. Suppose to the contrary that $x^*(q) = 0$ for some $q \in [0,1]$. Then $a^*(q) = q$. So $p^G(q) = 1$ and $\chi^B(q) = q$. So $V(q) = \lambda w(1) + \delta V(q)$. But when the firm has a bad project, payoff from giving a good rating is $w(1) > \frac{\lambda\delta}{1-\delta} w(1) = \delta V(q)$, where $\delta V(q)$ is the agency's payoff from giving a bad rating. So the strategic type agency has a profitable deviation to lie. Contradiction.

Therefore, lying must be a best response to the strategic type rating agency, for all $q \in [0,1]$. So the strategic type agency's equilibrium payoff is equal to his payoff if he lies for sure. When the firm has a good project, the agency gives a good rating, gets paid the rating fee, and reputation does not change since both types give a good rating for sure to a firm with a good project. When the firm has a bad project, When the firm has a bad project, by lying, the agency gets paid the rating fee, and loses all reputation and thus future payoff becomes 0. So

$$V(q) = w\left(\frac{\lambda}{\lambda + (1-\lambda)(1-a^*(q))}\right) + \lambda\delta V(q)$$

Rearranging,

$$V(q) = \frac{w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-a^*(q))}\right)}{1-\lambda\delta}. \tag{1}$$

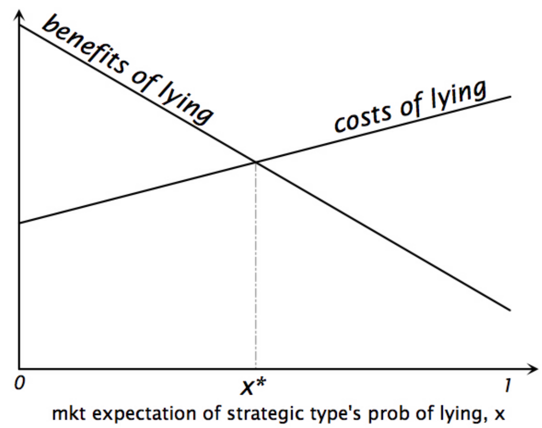
When $q = 1$, expected accuracy is $a^*(1) = 1 > a^*(q)$ for all $q < 1$ given that $x^*(q) > 0$. So $V(1) = \frac{w(1)}{1-\lambda\delta} > V(q)$ for all q. Because $w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-a)}\right)$ is continuous in a, is greater than $\delta V(1)$ for $a = 1$, and stricty increasing in a for $a > \underline{a}$, and equal to $0 < \delta V(1)$ for $a \leq \underline{a}$, there exists a unique solution a_1^M to $w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-a)}\right) = \delta V(1)$. Define $\rho_1^M = a_1^M$. For $q \in \left[\rho_1^M, 1\right]$,

$w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-a)}\right) \geq \delta V(1) \geq \delta V\left(\frac{q}{a}\right)$ for all $a \in [q,1]$ and strict inequality holds for all $a > q$. So lying is a best response for all market expected accuracy. So in equilibrium, the strategic type agency must lie with probability 1:

$$x^*(q) = 1, \text{and } a^*(q) = q, \text{ for all } q \in \left[\rho_1^M, 1\right].$$

Plugging this into (1), it follows that $V(q)$ increases strictly with q for $q \in \left(\rho_1^M, 1\right)$. Define a_2^M to be the unique solution to $w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-a)}\right) = \delta V\left(\rho_1^M\right)$ and $\rho_2^M = a_2^M a_1^M$. For $q \in [\rho_2^M, \rho_1^M)$, $\delta V\left(\frac{q}{a}\right)$ has a unique intersection $a = a^*(q)$ with $w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-a)}\right)$ because for $a = q$, $\delta V\left(\frac{q}{a}\right) = \delta V(1) = w\left(\frac{\lambda}{\lambda+(1-\lambda)\left(1-a_1^M\right)}\right) > w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-q)}\right)$, and for $a = \frac{q}{\rho_1^M} > a_2^M$, $w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-a)}\right) > w\left(\frac{\lambda}{\lambda+(1-\lambda)\left(1-a_2^M\right)}\right) = \delta V\left(\rho_1^M\right) = \delta V\left(\frac{q}{a}\right)$, and $\delta V\left(\frac{q}{a}\right)$ is well-defined and strictly decreasing in a for $a \in \left[q, \frac{q}{\rho_1^M}\right]$. Inductively define a_k^M to be the unique solution to $w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-a)}\right) = \delta V\left(\rho_{k-1}^M\right)$ and $\rho_k^M = \Pi_{j=1}^k a_j^M$. For $q \in [\rho_k^M, \rho_{k-1}^M)$, $\delta V\left(\frac{q}{a}\right)$ as a function of a and $w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-a)}\right)$ as a function of a has a unique intersection $a^*(q)$ because, for $a = q$, $\delta V\left(\frac{q}{a}\right) = \delta V(1) = w\left(\frac{\lambda}{\lambda+(1-\lambda)\left(1-a_1^M\right)}\right) > w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-q)}\right)$, and for $a = \frac{q}{\rho_{k-1}^M} > a_k^M$, $w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-a)}\right) > w\left(\frac{\lambda}{\lambda+(1-\lambda)\left(1-a_k^M\right)}\right) = \delta V\left(\rho_{k-1}^M\right) = \delta V\left(\frac{q}{a}\right)$, and $\delta V\left(\frac{q}{a}\right)$ is well-defined and strictly decreasing in a for $a = \left[q, \frac{q}{\rho_{k-1}^M}\right]$. Then, the equilibrium probability of lying $x^*(q)$ must satisfy $a^*(q) = 1-(1-q)\,x^*(q)$, so that the strategic type agency is indifferent between lying and not lying. We have shown that in equilibrium, a strategic type must lie with positive probability. If $x^*(q) = 1$, then $a^*(q) = q$, but then the benefits of lying are smaller than the costs of lying because $w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-q)}\right) < \delta V\left(\frac{q}{q}\right)$. So the agency's best response is to lie with probability 0, contradiction to $x^*(q) = 1$. For $x^*(q) \in (0,1)$ to be a best response, the strategic type must be indifferent between lying and not lying, and thus the equilibrium accuracy $a^*(q)$ must be pinned down by the intersection of the benefits of lying curve and the costs of lying curve. Moreover, $a^*(q)$ thus defined is continuous and strictly decreasing in q, because the curve $\delta V\left(\frac{q'}{a}\right)$ lies to the left of $\delta V\left(\frac{q''}{a}\right)$ if $q' < q''$ and $q', q'' \in [\rho_k^M, \rho_{k-1}^M)$. Therefore, by (1), $V(q)$ is strictly decreasing in q for $q \in [\rho_k^M, \rho_{k-1}^M)$.

We can now verify that the behaviors and beliefs posited is consistent with equilibrium. Since a bad rating is only given to a bad firm, and a firm always obtains a rating, $w\left(p^B\right) = w\left(p^\emptyset\right) = 0$ is consistent. A firm gets zero payoff whether he obtains a good rating, a bad rating, or no rating. Thus it is optimal to obtain a rating with probability 1. When the firm has a good project, giving the firm a good rating yields $w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-a^*(q))}\right) + \delta V(q) \geq \delta V\left(\frac{q}{a^*(q)}\right) + \delta V(q)$ because lying is a best response in equilibrium. So it is optimal for a strategic type agency to give a good rating to a good firm.



The reputation mechanism can be summarised by the graph.

What happens to this picture when there is a competitor agency 2? I will show that competition does not change the benefits of lying curve, but lowers the costs of lying curve, thereby increasing the

Theorem 1 *Consider* $\frac{\lambda\delta}{1-\delta} < 1$. *There exists a fee-pooling equilibrium under duopoly where* $V(q_1, q_2) > V(q_2, q_1)$ *for all* $q_1 > q_2$. *Comparing the market outcome of this equilibrium under duopoly at reputation profile* (q_1, q_2) *with market outcome in the MMR equilibrium under monopoly at reputation level* q_1,

1. *either a bad firm is given a good rating with probability* 1 *in both, or*

2. *a bad firm is given a good rating with a strictly higher probability under duopoly than under monopoly.*

2 Monopoly

I describe an equilibrium that exhibits the same behavior as in Mathis et al.

My monopoly model differs from that in Mathis et al.(2009) in that the agency sets its own fee and the firm may choose to obtain no rating. I will describe an equilibrium in which the market believes that the firm is bad for sure if the firm obtains no rating or a bad rating. Thus, if p^G is the market's belief about the firm if the firm obtains a good rating from the monopoly rating agency, then the firm's willingness-to-pay for a good rating is $w\left(p^G\right)$. The monopolist rating agency thus charges the firm its willingness-to-pay $w\left(p^G\right)$ in equilibrium.

I show that monopoly power implies that the agency will leave zero surplus to the firm and charge a fee equal to the value of the agency's good rating. Given this observation, I can then use similar arguments as under Mathis et al.(2009) to obtain the unique equilibrium.

I first focus on the opportunistic agency and the firm. I construct and show the uniqueness of their behaviors in equilibrium. Because I look at fee-pooling equilibria, I establish existence by checking that the fee the opportunistic agency charges is also optimal for the honest agency. Unless specified otherwise, an agency refers to the opportunistic agency.

In the monopoly case, I use notation without subscripts. For example, $p^G(q)$ is the investors' belief about a firm with a good rating from the monopolist with reputation q, and $x^*(q)$ is the probability an opportunistic monopolist lies when its reputation at the beginning of a given period is q.

2.1 Preliminaries

In this section, I show that an opportunistic agency's cost of lying is its discounted future value after giving a bad rating, $\delta V\left(\chi^B\right)$, because its future value drops to zero once its reputation drops to zero. I also show that an opportunistic agency's benefits of lying is the value of its good rating, $w\left(p^G\right)$, by establishing that a firm gets financed if and only if it receives a good rating, and thus an agency gets paid only if it issues a good rating.

We first observe that a monopolist known to be opportunistic has zero value because it has an incentive to give every firm a good rating to ensure that securities are issued and the agency is paid. But this agency is unable to provide any information, and so securities are not issued even if a firm gets a good rating.

Lemma 2 *An agency with zero reputation has zero value:* $V(0) = 0$.

Next, we see that the firm will get financed if and only if it obtains a good rating. The proofs are in the Appendix.

Lemma 3 *Only a good rating has positive value. That is,* $w\left(p^G(q)\right) > 0$ *for all* $q > 0$*, but* $w\left(p^B(q)\right) = w\left(p^\emptyset(q)\right) = 0$ *for all* $q \geq 0$.

Because a bad rating has zero value, and thus no securities will be issued for it, an agency gets paid if and only if it gives a good rating. Therefore, the only valuable "good" the agency can sell to a firm is its good rating. The cost of this "good" is

$$c^v = \delta V\left(\chi^B\right) - \delta V\left(\chi^{Gv}\right)$$

when the firm's project is of quality v. If the agency publishes a good rating, then the project will be financed, and its quality is observed ex post. Thus, the agency's reputation at the beginning of next period is χ^{Gv}. If the agency publishes a bad rating instead, the project will not be financed, and its quality not observed; thus the agency's reputation at the beginning of the next period becomes χ^B regardless of firm type. The cost for the agency to provide a good rating to a type v firm is the difference in the discounted value of its future reputation from publishing a good rating instead of a bad one. Then, the agency's cost of lying is

$$c^b = \delta V\left(\chi^B\right)$$

because lying is always detected, $\chi^{Gb} = 0$, and $V\left(\chi^{Gb}\right) = 0$ by lemma 2.

It follows immediately that issuing a good rating is cheaper for a good firm than for a bad firm:

$$c^g < c^b, \tag{2}$$

because $\chi^{Gg} > 0$ and thus $V\left(\chi^{Gg}\right) > 0$ by the *active* condition.

Because obtaining a bad rating or no rating both give a bad rating both give the firm zero payoff, a firm's willingness-to-pay for the agency's good rating is the value of investors' belief p^G, i.e., $w\left(p^G\right)$. Because a monopolist will leave zero surplus, it will charge

$$\phi^v = w\left(p^G\right) \tag{3}$$

for both types of firms. Thus the agency's benefit from lying is the value of its good rating.

It follows from (2) that the surplus from trade, $w\left(p^G\right) - c^v$, is higher when the firm is good. Because the monopolist captures the entire surplus from trade and has a positive value, there must be positive surplus from trade with a good firm, and a good firm will obtain a good rating with probability 1 (lemma 4).

Lemma 4 *There is positive surplus to provide a good rating to a good firm:* $w\left(p^G\right) > c^g$*; an opportunistic agency gives a good rating to a good firm for sure:* $y^*(q) = 1$ *for all* $q > 0$*, and a good firm obtains a rating with probability 1:* $\alpha^*(q) = 1$*.*

2.2 Equilibrium Probability of Lying

In this section, I construct and show the uniqueness of equilibrium by determining an opportunistic agency's equilibrium probability of lying $x^*(q)$ and showing that a bad firm obtains a rating for sure.

By lemma 4, a good firm obtains a good rating for sure. A bad firm obtains a good rating if it approaches the agency, which it does with probability $\beta^*(q)$, and if the agency gives it a good rating, which occurs with probability

$$1 - a^*(q) = (1-q)\, x^*(q).$$

Following Mathis et al.(2009), I define $a^*(q)$ as the agency's equilibrium accuracy. Thus, the investors believe a project with a good rating to be good with probability

$$p^G = \frac{\lambda}{\lambda + (1-\lambda)\,\beta^*(q)\,(1-a^*(q))}. \tag{4}$$

Thus, the value of the agency's good rating, $w\left(p^G\right)$, which is also its benefits of lying, is strictly increasing in its equilibrium accuracy.

Because a bad rating is only given to a bad firm, after issuing a bad rating, the agency's reputation becomes

$$\chi^B(q) = \frac{q}{a^*(q)}.$$

Therefore, the equilibrium probability of lying $x^*(q)$ must satisfy

$$x^*(q) = \begin{cases} 1 & \text{if } w\left(\frac{\lambda}{\lambda+(1-\lambda)\beta^*(q)(1-a^*(q))}\right) \geq \delta V\left(\frac{q}{a^*(q)}\right) \\ \in [0,1] & \text{if } w\left(\frac{\lambda}{\lambda+(1-\lambda)\beta^*(q)(1-a^*(q))}\right) = \delta V\left(\frac{q}{a^*(q)}\right) \\ 0 & \text{if } w\left(\frac{\lambda}{\lambda+(1-\lambda)\beta^*(q)(1-a^*(q))}\right) \leq \delta V\left(\frac{q}{a^*(q)}\right) \end{cases} . \tag{5}$$

We now derive the Bellman equation for the value function. Because a good firm always obtains a good rating, reputation remains unchanged after a good rating is issued for a good firm: $\chi^{Gg}(q) = q$. If an opportunistic agency never lies, then its reputation never changes, and it receives its rating fee equal to the value of its good rating, $w(1)$, only when the firm has a good project. Thus, its equilibrium payoff would be

$$V(q) = \lambda (w(1)) + \delta V(q).$$

Thus, an equilibrium exists in which an opportunistic agency never lies when reputation is q only if

$$w(1) \leq \delta V\left(\chi^B(q)\right) = \delta V(q) = \frac{\delta\lambda}{1-\delta} w(1). \tag{6}$$

Proposition 5 *If $\frac{\delta}{1-\lambda\delta} \geq 1$, then in the unique equilibrium, the agency never lies, i.e., $x^*(q) = 0$ for all $q > 0$.*

Proof. Proposition 5 is a direct application of Proposition 1 in Mathis et al.(2009) because the fee the agency receives in equilibrium is determined by p^G according to an exogenous function increasing in p^G. ■

On the other hand, when $\frac{\delta}{1-\lambda\delta} < 1$, inequality (6) cannot hold. Thus, an opportunistic agency lies with positive probability at every reputation level $q \in [0,1]$. Thus it must weakly prefer lying, and thus its equilibrium payoff is equal to its payoff if it always lies for a bad firm:

$$V(q) = w\left(p^G(q)\right) + \lambda\delta V(q) + (1-\lambda)\delta V(0).$$

It follows that

$$V(q) = \frac{w\left(p^G(q)\right)}{1-\lambda\delta}. \tag{7}$$

Proposition 6 *If $\frac{\delta}{1-\lambda\delta} < 1$, then in the unique equilibrium, an opportunistic agency always lies: $x^*(q) > 0$, and $V(q)$ is continuous and strictly increasing in q, for all $q \in [0,1]$.*

Proof. If a monopolist does not rate a firm, its reputation does not change. The opportunistic agency prefers rating a bad firm to not doing so because $w\left(p^G(q)\right) > \frac{\delta}{1-\lambda\delta} w\left(p^G(q)\right) = \delta V(q)$ by (7). By *maximum coverage* condition, $\beta^*(q) = 1$ for all $q > 0$. We have thus pinned down all behaviors except $x^*(q)$ to be the same as under Mathis et al.(2009). It follows that the equilibrium probability of lying, $x^*(q)$, must be the unique function constructed in Mathis et al.(2009).

I establish existence by showing that the rating fee given by (3) is optimal for an honest agency as well. When the firm has a good project, future reputation remains q whether the agency rates it or not, but the agency receives a positive rating fee by doing so. When the firm has a bad project, an agency can earn a higher reputation $\chi^B(q) > q$ by giving this firm a bad rating than by not rating the firm. Because the equilibrium rating fee increases in the agency's reputation, and thus future payoffs increase in the future reputation regardless of agency type, both types of the agency strictly prefer rating a bad firm to not doing so. The optimality of the rating fee (3) follows immediately ■

I outline the construction of $x^*(q)$ here. Define ρ_1^M to be the unique solution to

$$w\left(\frac{\lambda}{\lambda + (1-\lambda)\left(1-\rho_1^M\right)}\right) = \frac{\delta}{1-\lambda\delta} w(1) = \delta V(1). \tag{8}$$

In the unique equilibrium, $x^*(q) = 1$, and thus

$$V(q) = \frac{w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-q)}\right)}{1-\lambda\delta},$$

for all $q \geq \rho_1^M$ by (4), (7) and because $\beta^*(q) = 1$. For all $q < \rho_1^M$, the agency must be indifferent between lying and not lying in equilibrium, and $a^*(q)$ is the solution a^* to

$$w\left(\frac{\lambda}{\lambda + (1-\lambda)(1-a^*)}\right) = \delta V\left(\frac{q}{a^*}\right).$$

The equilibrium accuracy $a^*(q)$, and thus equilibrium probability of lying $x^*(q)$, can thus be solved backward from $q \geq \rho_1^M$.

3 Duopoly

We now consider competition between two long-lived rating agencies with an identical discount factor $\delta \in (0,1)$. Let $q = (q_1, q_2)$ denote their reputation profile at the beginning of a given period. Because we focus on symmetric equilibria, every equilibrium is associated with a value function $V(q_i, q_j) = V_i(q)$, which is the opportunistic agency i's equilibrium payoff when its own reputation is q_i, and its competitor's reputation is q_j.

When the two agencies have different reputation levels, we call the one with the superior reputation the "leading agency" and the one with the inferior reputation the "trailing agency." I focus here on equilibria where the leading agency has a higher value of business, i.e., $V(q_1, q_2) > V(q_2, q_1)$, if $q_1 > q_2$. I discuss the implications of this assumption in Section 4. Omitted proofs are in the Appendix.

The question is whether competition can improve the quality of information. My focus is thus the case where $\frac{\delta}{1-\lambda\delta} < 1$ because, when $\frac{\delta}{1-\lambda\delta} \geq 1$, the monopoly already achieves the first best.

3.1 Preliminaries

We first establish that, as under monopoly, an agency known to be opportunistic has zero value under duopoly. Thus an agency's future value of business drops to 0 once it lies. In addition, a firm will not be financed if it obtains no rating or a bad rating. I then show that an opportunistic agency lies with positive probability.

Lemma 7 *An agency with zero reputation has zero payoff:* $V(0,r)=0$ *for all* $r\in[0,1]$*. In addition, for all* $q\in[0,1]^2$*, a bad rating or no rating has zero value:* $w\left(p^{\emptyset}(q)\right)=w\left(p^{B_i}(q)\right)=0$ *for* $i\in\{1,2\}$*.*

Therefore, as in monopoly case, the only "good" an agency sells is its good rating. A firm's willingness-to-pay for agency i's good rating is $w\left(p^{G_i}\right)$. The cost for agency i to provide a good rating to a firm with a type v project is

$$c_i^g = \delta V\left(\chi^{B_i}\right)-\delta V\left(\chi^{G_i g}\right) \quad (9)$$

$$c_i^b = \delta V\left(\chi^{B_i}\right)-\delta V\left(0,q_{-i}\right)=\delta V\left(\chi^{B_i}\right). \quad (10)$$

Thus, competition does not change the reputation mechanism that trades off the current rating fee with the future value of reputation χ^{B_i}.

We first see that, as under monopoly, an opportunistic agency i lies with positive probability.

Lemma 8 *An opportunistic agency* i *lies with positive probability,* $x_i^*(q)>0$*, at every reputation profile* $q\in[0,1]^2$*, for* $i=1,2$*.*

Proof. Because the equilibrium is *active*, $\lambda_i>0$. Suppose to the contrary that $x_i^*=0$, then $p^{G_i}=\frac{\lambda_i}{\lambda_i+(1-\lambda_i)(1-q_i)x_i}=1$ and $\chi_i^{B_i}=\frac{q_i}{q_i+(1-q_i)x_i}=q_i$. Because $\frac{\delta}{1-\lambda\delta}<1$, the maximum payoff that agency i can hope for at any reputation profile is to rate the firm, whatever its type, at a fee equal to the maximum possible value $w(1)$. Thus, for all $q'\in[0,1]^2$,

$$V_i\left(q'\right)\leq\frac{\delta}{1-\lambda\delta}w(1)<w(1). \quad (11)$$

Hence, agency i's payoff from not rating the bad firm is strictly smaller than $w(1)$. Because i is providing zero surplus to a bad firm in equilibrium, its competitor must also provide zero surplus to this firm. Thus, a bad firm will approach agency i for sure if i charges $w(1)-\varepsilon$ for a sufficiently small $\varepsilon>0$. But then i has a profitable deviation by charging $w(1)-\varepsilon$ to a bad firm, contradiction. ■

3.2 Main Result

The main result of this paper compares the market outcomes under duopoly at reputation profile (q_1,q_2) where $q_1\geq q_2$, with the market outcome when a monopolist has reputation q_1.

I use two criteria to judge efficiency. Since it is inefficient to invest in a bad project, the first criteria is the probability that a bad firm is financed, that is, $\sum_{i\in I}\beta_i^*(q)\,a_i^*(q)$. Since a firm's financial market payoff reflects the amount of investment it receives, the second criteria is the value of a good rating that a good firm receives: $w\left(p^{G_i}\right)$. I show that adding a competitor with a lower or equal reputation worsens market efficiency according to both criteria.

Say that agency i is active if it rates a firm with positive probability in the current period, i.e., if $\alpha_i(q)+\beta_i(q)>0$.

Theorem 9 *Consider* $\frac{\delta}{1-\lambda\delta}<1$. *There exists an equilibrium under duopoly where the leading agency has a higher value, i.e.* $V(q_1,q_2)>V(q_2,q_1)$, *for all* $0\leq q_2<q_1\leq 1$. *Comparing the market outcome at a reputation profile* (q_1,q_2) *in such an equilibrium under duopoly and the market outcome at reputation level* q_1 *in the unique equilibrium under monopoly, where* $0\leq q_2\leq q_1\leq 1$, *then either*

1. *efficiency is the same, and an active agency lies for sure under both market structures:* $x_i^*(q_1,q_2)=x^*(q_1)=1$ *for active agency* i, *or*

2. *under duopoly, efficiency is worse, and an active agency lies more often:* $x_i^*(q_1,q_2)>x^*(q_1)$.

I will show that if a monopolist rating agency would lie with probability 1, then it would still lie with probability 1 if it faced competition. If a monopolist agency randomizes between lying and not lying, the equilibrium probability of lying is determined by the intersection of the benefits-from-lying curve and the cost-of-lying curve. Competition increases equilibrium probability of lying because it has no effect the benefits-of-lying curve, but lowers the cost-of-lying curve by lowering future value of business (figure 1).

In proving the theorem, the first step (lemma 10, Section 3.3) involves showing that if an agency rates a firm of type v with positive probability in equilibrium, then when a type v firm approaches, the agency either gives it a good rating with probability 1 or leaves zero surplus. Therefore, if an agency randomizes between giving a good or bad rating to a bad firm, then it must charge a fee equal to the value of its good rating. Thus, as under monopoly, current benefits from lying for an active agency under duopoly is still equal to the value of the agency's good rating, which is $w\left(p^G\right)=w\left(\frac{\lambda_i}{\lambda_i+(1-\lambda_i)(1-q_i)x_i^*}\right)$, where λ_i is probability that a firm rated by agency i is good.

The second step (lemma 13, Section 3.4) is to show that when two agencies have different reputations at the beginning of a given period, then only the leading agency is active in that period. Therefore, when the value of the leading agency's good rating is sufficiently high such that it strictly prefers rating a bad firm, by *maximum coverage*, a bad firm obtains a rating for sure, and $\lambda_i=\lambda$. Thus, competition does not affect the composition of the firms being rated. The first two steps imply that the benefits-of-lying curve does not change with competition.

The final step (lemma 14, Section 3.5) is to show that competition reduces an agency's value of business, and thus lowers the cost-of-lying curve in figure

1, because competition forces agencies to leave positive surplus for firms with good projects.

It follows from these three steps that the leading agency lies with higher probability when faced with competition (figure 1). In Section 3.5, I also show that equilibrium accuracy and market efficiency are both lower under duopoly.

Lastly, I establish existence by construction in Section 3.6.

3.3 Surplus and Probability of Trade

In this section, I establish the key observation that leads to Theorem 9: competition does not cause an agency to give a positive surplus to a bad firm, as long as the agency does not lie with probability 1. This observation follows from the zero surplus lemma saying that agency i will give a good rating to a type v firm for sure if there is positive surplus from trade between this firm and agency i. I then show that, as under monopoly, an agency gives a good rating to a good firm for sure. It then follows that a duopolist's equilibrium probability of lying is still determined by balancing the future value of reputation from issuing a bad rating, and the value of its good rating.

Lemma 10 (Zero Surplus) *If there is positive surplus from trade between agency i and a type v firm: $w\left(p^{G_i}\right) > c_i^v$, then agency i will give this firm a good rating for sure: $x_i^* = 1$ if $v = b$ and $y_i^* = 1$ if $v = g$, for $i = 1, 2$.*

At the fee-setting stage, the two agencies compete for business by giving a bigger surplus to the firm. Because a firm has a zero payoff whether it has no rating or a bad one, when the firm has a good project, the surplus it gets from approaching agency i at fee ϕ_i is

$$U_i^g(q, \phi_i) := (q_i + (1 - q_i)\, y(\phi_i; q)) \left(w\left(p^{G_i}\right) - \phi_i\right);$$

when it has a bad project, the surplus it gets is

$$U_i^b(q, \phi_i) := (1 - q_i)\, x(\phi_i; q) \left(w\left(p^{G_i}\right) - \phi_i\right).$$

I now prove lemma 10. Suppose to the contrary that $w\left(p^{G_i}\right) > c_i^v$ and that the opportunistic agency i randomizes strictly between giving a good and bad rating to a type v firm in equilibrium. Because i randomizes strictly, i must be charging a type v firm at cost c_i^v and thus leaves this firm a positive surplus. Bertrand competition implies that agency $-i$ must be leaving this firm an equal amount of surplus in equilibrium. Then, by charging $\phi_i' = c_i^v + \varepsilon$ for a sufficiently small $\varepsilon > 0$, agency i can leave a bigger surplus to the type v firm, because the probability that agency i gives a good rating to a type v firm jumps upward to 1 but the firm's surplus conditional on obtaining a good rating from i decreases continuously with ϕ_i. Thus, by raising the rating fee slightly above cost, agency i will get the business of a type v firm with probability 1, and earn a higher fee conditional on rating the firm. Because both agencies leave equal and positive amount of surplus for this firm in equilibrium, agency

i must weakly prefer rating this firm at its equilibrium fee $\phi_i^v = c_i^v$ to letting its competitor rate it. Otherwise, if i rates this firm with positive probability, i has a profitable deviation by leaving negative surplus so that this firm will approach $-i$ for sure; if i does not rate this firm, then charging its equilibrium fee is weakly dominated by charging $w\left(p^{G_i}\right) - \delta$ for sufficiently small δ. But, if i weakly prefers rating the firm at the equilibrium fee $\phi_i^v = c_i^v$, charging $c_i^v + \varepsilon$ is then a profitable deviation for i, a contradiction. We have thus proven lemma 10.

By the zero surplus lemma, if agency i does not lie for sure, it must give zero surplus to a bad firm by charging the firm its willingness-to-pay: $\phi_i^v = w\left(p^{G_i}\right)$. It seems puzzling that the firm, the buyer of a good rating, gains no positive surplus when there is greater competition. However, the reason the firm receives zero surplus when the agency is randomizing is that the value of the agency's good rating is equal to the agency's cost to produce it. Thus, the agency also receives zero surplus from selling a good rating to the firm. So, the reason a firm does not get a positive surplus under competition is that there is zero surplus from trade to begin with. Hence, no matter which party has the bargaining power, the surplus for each remains zero. Thus, this result does not depend on the drastic change in the distribution of bargaining power due to the Bertrand competition model. We can also model competition as reducing the agency's share of the surplus from 1 to some $\kappa \in (0,1)$, and the result will remain unchanged.

Lemma 10 is very general. It does not depend on the condition that the leading agency has a higher value than the trailing agency. It goes through even if there is imperfect monitoring and an agency's reputation never drops to zero.

Using the zero surplus lemma, I can then show that under duopoly, an agency will give a good rating to a good firm for sure. I do so by first showing that there is nonnegative surplus from trade between a bad firm and an agency (lemma 11), and thus there is positive surplus from trade between a good firm and an agency because cost of a good rating is lower for a good firm.

Lemma 11 *$w\left(p^{G_i}\right) \geq c_i^b$ for all $q \in (0,1)^2$, for $i = 1,2$.*

Proof. Suppose to the contrary that $w\left(p^{G_i}\right) < c_i^b$. Then, agency i will not give a bad firm a good rating at any fee a bad firm is willing to accept. Then $p^{G_i} = 1$ by Bayes' rule if agency i is active, and by *rating-consistency* if agency i is inactive. Hence, $w\left(p^{G_i}\right) = w(1) > \frac{\delta}{1-\lambda\delta} w(1) \geq \delta V_i(q')$ for any reputation profile q'. We have obtained a contradiction because $c_i^b = \delta V_i\left(\chi^{B_i}\right)$. ■

Lemma 12 follows by the zero surplus lemma and by noting that it is cheaper to give a good rating to a good firm than to a bad firm, because $\delta V\left(\chi^{G_i g}\right) > 0 = \delta V(0, q_{-i})$ for all $q_i > 0$ (lemma 7 and *active* equilibrium).

Lemma 12 *An agency gives a good rating to a good firm for sure: $y_i^*(q) = 1$, for $i = 1,2$, and $q \in (0,1]^2$.*

Therefore, only a bad firm is given a bad rating. Thus, when a good rating is given to a good firm, reputation profile is unchanged:

$$\chi^{G_i g}(q_i, q_j) = (q_i, q_j); \tag{12}$$

after giving a bad rating, agency i's reputation becomes

$$\chi_i^{B_i}(q) = \frac{q_i}{a_i^*(q)},$$

where $a_i^*(q) := 1 - (1 - q_i)\, x_i^*$ is the probability that agency i gives a bad rating to a bad firm in equilibrium, which is defined as agency i's equilibrium accuracy. Then, the value of agency i's good rating is

$$w\left(p^{G_i}\right) = w\left(\frac{\lambda_i}{\lambda_i + (1 - \lambda_i)(1 - a_i^*)}\right).$$

Thus the only way that competition can change the benefits-of-lying curve is by affecting the mix of firms seeking a rating from agency i.

3.4 Only the Leading Agency is Active

In this section, I show that in an equilibrium where the leading agency has a higher value, only the leading agency issues a rating. We label the leading agency as agency 1 without loss of generality.

Lemma 13 (active agency) *In an equilibrium where the leading agency has higher value,* $\alpha_1^*(q) = 1$ *and* $\beta_2^*(q) = 0$ *for all* $0 \leq q_2 < q_1 \leq 1$.

The key battle is over who wins a good firm's business. When the firm is good, future reputation profile remains the same no matter who rates it, but an agency gets paid only by rating it. Thus, an agency strictly prefers to rate a good firm and will compete for business by giving a larger surplus to the firm.

Because an opportunistic agency i gives a good rating to a good firm for sure in equilibrium (lemma 12), its equilibrium fee must be at least equal to its cost c_i^g. Thus the maximal surplus that agency i can leave a good firm in equilibrium is

$$\overline{U}_i^g := w\left(p^{G_i}\right) - c_i^g = w\left(p^{G_i}\right) - \delta V\left(\chi_i^{B_i}, q_{-i}\right) + \delta V_i(q). \tag{13}$$

If both agencies randomize strictly between lying and not lying in equilibrium, then by the zero surplus lemma, $w\left(p^{G_i}\right) = \delta V_i\left(\chi^{B_i}\right)$ for $i = 1, 2$. By (13) and the condition that the leading agency has higher value, the leading agency is able to leave a bigger surplus for a good firm and thus will win all of its business. Thus, the trailing agency must be inactive, otherwise $\lambda_2(q) = 0$, contradicting the *active* condition. In Appendix A.3, I discuss the other cases.

3.5 Value of Reputation

This section shows that competition decreases the value of reputation for both the leading and trailing agency. Here $V(.,.)$ refers to the value function under duopoly and $V(.)$ refers to the value function under monopoly.

Lemma 14 *In an equilibrium in which the leading agency has a higher value,* $V(q_1, q_2) \leq V(q_1)$ *for all* $(q_1, q_2) \in [0,1]^2$*, and strict inequality holds for all* $(q_1, q_2) \in (0,1)^2$*.*

It suffices to discuss the case in which $q_1 > q_2$ because when the two agencies have equal reputations, by symmetry, each agency plays the role of a leading agency and trailing agency with probability $\frac{1}{2}$.

We first consider the case in which the leading agency is believed to be honest. The proof does not rely on the condition that the leading agency has higher value.

Lemma 15 $V(1,r) = V(1)$ *and* $V(r,1) = 0$ *for all* $r \in [0,1)$*. In addition, a good firm approaches the agency with a perfect reputation with probability* 1*.*

Proof. Because agency 1 is believed to be honest, it can leave a surplus up to $w(1)$ to a good firm.

I first argue that $\lambda_2 < 1$. Suppose to the contrary, then $w\left(p^{G_1}\right) = w\left(p^{G_2}\right) = w(1)$, then Bertrand competition will drive a good firm's fee down to zero. But then $V(r,1) = 0$ because a good firm pays zero rating fee, but a bad firm never approaches agency 2, and reputation profile never change. In addition, when the firm is bad, agency 2 can get its business by charging a fee slightly below $w(1)$, because the bad firm expects zero surplus from agency 1, and an opportunistic agency 2 will give a good rating at such a fee given that $w(1) > \delta V(q')$ for any q'. Agency 2 strictly prefers doing so to not rating the firm and obtaining $V(r,1) = 0$ in the future, a contradiction to $\lambda_2 = 0$.

Hence, $\lambda_2 < 1$, and thus $w\left(p^{G_2}\right) < w(1)$ because $x_2^* > 0$ by lemma 8. Thus a good firm never approaches agency 2 in equilibrium. But then neither does a bad firm, otherwise $\lambda_2 = 0$. So, agency 2 is inactive, and thus $V(r,1) = 0$ because agency 1 never loses its reputation.

We then see that there must be zero surplus from trade between agency 2 and a bad firm, i.e. $w\left(p^{G_2}\right) = \delta V_2\left(\chi^{B_2}\right)$, otherwise a bad firm will obtain a rating from agency 2 for sure, but then $\lambda_2 = 0$. Thus, by (13), agency 2 can leave only a zero surplus to a good firm. Hence, $\phi_1^g = w(1)$. An opportunistic agency 1 will then obtain $V(1,r) = \frac{w(1)}{1-\lambda\delta} = V(1)$. ■

Because the trailing agency is inactive until the leading agency lies and loses all of its reputation, it derives value from becoming a monopolist at the current reputation level at a future date. Thus, its value is lower than being a monopolist today. But, as long as $0 < q_2 < q_1 < 1$, the trailing agency becomes a monopolist with positive probability, and thus derive positive value.

Although trailing agency 2 is inactive, by lemma 11 and (13), agency 2 can leave a good firm a surplus equal to at least $\delta V_2(q)$. To compete for business,

$$\phi_1^g \leq w\left(p^{G_1}\right) - \delta V_2(q) < w\left(p^{G_1}\right) \tag{14}$$

for all $0 < q_2 < q_1 < 1$. This discount to a good firm reduces value for the leading agency.

Claim 16 *When $q_1 \geq \rho_1^M$ and $q_2 < q_1$, then $x_1^*(q) = 1$ and $\beta_1(q) = 1$, and thus $w\left(p^{G_1}(q)\right) = w\left(p^G(q_1)\right)$, but $V(q_1, q_2) \leq V(q_1)$, and strict inequality holds if $q_1 < 1$.*

Proof. Agency 1 must lie for sure in equilibrium, otherwise,

$$\begin{aligned} w\left(p^{G_1}\right) &= w\left(\frac{\lambda}{\lambda + (1-\lambda)\beta_1(q)(1-q_1)x_1^*}\right) \\ &> w\left(\frac{\lambda}{\lambda + (1-\lambda)\left(1-\rho_1^M\right)}\right) \\ &= \delta V(1) \geq \delta V_1\left(\chi^{B_1}\right) \end{aligned}$$

by (8) and because $w(.)$ is strictly increasing. Thus, $x_1^* = 1$ by the zero surplus lemma. Because the surplus from a good rating between agency 1 and a bad firm is positive, in equilibrium, a bad firm must approach agency 1 with probability 1. Thus, the value of agency 1's good rating is $w\left(p^{G_1}\right) = w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-q_1)}\right)$, which is the same as when agency 1 is a monopolist. It is then immediate from (14) that the leading agency's value of business goes down as long as $0 < q_2 < q_1 < 1$. ■

However, the value of business for $q_1 \geq \rho_1^M$ is exactly the cost of lying when the leading agency's reputation is somewhat below ρ_1^M, which implies that for $q_1 < \rho_1^M$, the cost-of-lying curve shifts down. The following claim completes the proof for both lemma 14 and Theorem 9.

Claim 17 *$w\left(p^{G_1}(q)\right) < w\left(p^G(q_1)\right)$ and $V(q_1, q_2) \leq V(q_1)$ for all $0 \leq q_2 < q_1 < \rho_1^M$,*

We first observe that $w\left(p^{G_1}\right) = \delta V_1\left(\chi^{B_1}\right)$, otherwise $w\left(p^{G_1}\right) > \delta V_1\left(\chi^{B_1}\right)$ by lemma 11, but then $x_1^* = 1$ and $\delta V_1\left(\chi^{B_1}\right) = \delta V(1) > w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-q_1)}\right)$ because $q_1 < \rho_1^M$, which is a contradiction. Thus, agency 1 must charge a bad firm its willingness-to-pay.

Suppose to the contrary that there exists $0 < q_2 < q_1$ such that $V(q_1, q_2) \geq V(q_1)$. We can then (see Appendix A.4 for details) show that there exists $\hat{q} = (\hat{q}_1, \hat{q}_2)$ and $\overline{q}_1 \in \left(\hat{q}_1, \frac{\hat{q}_1}{\rho_1^M}\right)$ such that $V(\hat{q}_1, \hat{q}_2) \geq V(\hat{q}_1)$, but $V(q_1, q_2) < V(q_1)$ for all $q_2 < q_1$ and $q_1 > \overline{q}_1$ and for all $q_1 = \hat{q}_1$ and $q_2 \geq \chi_2^{B_2}(\hat{q})$.

If a bad firm obtains a rating for sure, then the benefits of lying as a function of equilibrium accuracy a_1^* are the same as under monopoly. But the cost of lying as a function of a_1^* is $\delta V\left(\frac{\hat{q}_1}{a_1^*},\hat{q}_2\right) < \delta V\left(\frac{\hat{q}_1}{a_1^*}\right)$ for all $a_1^* \leq \rho_1^M$ by construction of $\overline{q}_1$. Because for all $a_1 > \rho_1^M$, $w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-a_1)}\right) > \delta V(1) \geq \delta V\left(\frac{\hat{q}_1}{a_1},\hat{q}_2\right)$, the solution a_1^* to

$$w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-a_1^*)}\right) = \delta V\left(\frac{\hat{q}_1}{a_1^*},\hat{q}_2\right)$$

must be lower than the equilibrium accuracy $a^*(q_1)$ under monopoly, and as a result, the value of a good rating at $(\hat{q}_1,\hat{q}_2)$ must be lower than $w\left(p^G(\hat{q}_1)\right)$. Then,

$$\begin{aligned} V(\hat{q}_1,\hat{q}_2) &= \frac{\lambda\phi_1^g(\hat{q}) + (1-\lambda)\phi_1^b(\hat{q})}{1-\lambda\delta} \\ &< \frac{w\left(p^{G_1}(\hat{q})\right)}{1-\lambda\delta} \\ &< \frac{w\left(p^G(\hat{q}_1)\right)}{1-\lambda\delta} = V(\hat{q}_1). \end{aligned}$$

If a bad firm randomizes between obtaining a rating and no rating, by *maximum coverage* condition, the leading agency must weakly prefer that the trailing agency rates a bad firm. Therefore, the value of the leading agency's good rating is no more than the value that agency 1 derives from waiting and maintaining the same reputation, hoping that the trailing agency will lie so that it becomes a monopolist at the same reputation level in the future; that is,

$$\begin{aligned} w\left(p^{G_1}\right) &\leq (1-a_2^*(\hat{q}))V(\hat{q}_1) + a_2^*(\hat{q})V\left(\hat{q}_1,\chi_2^{B_2}(\hat{q})\right) \\ &< V(\hat{q}_1) \end{aligned}$$

because $a_2^*(\hat{q}) > 0$ by 8 and the construction of $\hat{q}_2$. Thus, the value of agency 1's good rating under duopoly is lower than that when it is a monopolist because $\frac{\delta}{1-\lambda\delta} < 1$.

3.6 Construction of Equilibrium

I show the existence of equilibria by construction. I first construct the leading agency's probability of lying function assuming that a bad firm obtains a rating for sure at all reputation profiles. We can then check, whether at some reputation profiles, this assumption contradicts the requirement that the leading agency has a higher value than the trailing agency. If so, then we need to modify the probability a bad firm obtains a rating. The value function associated with the equilibrium so constructed is continuous except possibly when the two agencies have an equal reputation. In addition, the value for the leading agency is strictly increasing in its own reputation, whereas that for the trailing agency is strictly decreasing in the leading agency's reputation. I construct behaviors off the equilibrium path and verify all equilibrium conditions in Appendix A.5.3 and A.5.4.

By symmetry, an equilibrium can be described by the strategy profile $\left(\phi^g, \phi^b, \alpha, \beta, x, y\right)$ together with the belief system

$$\left(\gamma^g, \gamma^b, \lambda, p^G, p^B, p^\emptyset, \chi^B, \chi^{Gg}, \chi^{Gb}\right)$$

which is a function of $q = (q_i, q_{-i})$ where the first argument is the agency's own reputation and the second argument is its competitor's reputation. For example, $p^G(q_i, q_j)$ is the probability a firm with agency i's good rating is believed to be good, when agency i's reputation is q_i and agency j's reputation is q_j. In the following, q_1 is taken to be the leading agency's reputation, and q_2 the trailing agency's reputation.

3.6.1 Preliminaries

I summarize the behaviors already pinned down by equilibrium.

By the active agency lemma 13 and by symmetry, at the equilibrium fee profile, a good firm obtains a rating for sure from the leading agency, and randomizes equally between the two when they have equal reputation. A bad firm does not approach the trailing agency: $\beta^*(q_i, q_j) = 0$ if $q_i < q_j$. By lemma 12, $y^*(q_i, q_j) = 1$.

Given equilibrium behaviors (x^*, β^*), agency i's accuracy is

$$a^*(q_i, q_j) = 1 - (1 - q_i)\, x^*(q_i, q_j)\,. \tag{15}$$

Then

$$p^G(q_1, q_2) = \frac{\lambda}{\lambda + (1 - \lambda)\, \beta^*(q_1, q_2)\,(1 - a^*(q_1, q_2))}$$

and

$$p^G(q_2, q_1) = \frac{\lambda(q_2, q_1)}{\lambda(q_2, q_1) + (1 - \lambda(q_2, q_1))\,(1 - a^*(q_2, q_1))}.$$

The reputation profile after i gives a good rating is $\chi^{Gg}(q_i, q_j) = (q_i, q_j)$, whereas that after it gives a bad rating is

$$\chi^B(q_i, q_j) = \left(\frac{q_i}{a^*(q_i, q_j)}, q_j\right)$$

for $i = 1, 2$.

We now derive the Bellman equation for the agencies' value functions. The reputation profile does not change if no rating is issued or a good firm is rated. If the leading agency lies for a bad firm, agency 2 becomes a monopolist at its current reputation q_2. Hence,

$$\begin{aligned} V(q_2, q_1) \quad = \quad & \frac{(1 - \lambda)\, \beta^*(q_1, q_2)\, \delta}{1 - \lambda\delta - (1 - \lambda)\,\delta\,(1 - \beta^*(q_1, q_2))} \\ & \times \left[(1 - a^*(q_1, q_2))\, V(q_2) + a^*(q_1, q_2)\, V\left(q_2, \frac{q_1}{a^*(q_1, q_2)}\right)\right] \end{aligned} \tag{16}$$

In equilibrium, the leading agency always weakly prefers to lie. Thus, its equilibrium payoff is equal to its payoff when it gives a good rating to both types of firm for sure:

$$V(q_1, q_2) = \frac{(\lambda + (1-\lambda)\beta^*(q_1, q_2))\, w\left(p^G(q_1, q_2)\right) - \lambda\delta V(q_2, q_1)}{1 - \lambda\delta - (1-\lambda)\delta(1-\beta^*(q_1, q_2))}. \tag{17}$$

I will construct an equilibrium in which a good rating has no positive surplus between a bad firm and a trailing agency; that is, $\delta V\left(\chi^B(q_2, q_1)\right) = w\left(p^G(q_2, q_1)\right)$ for all $q_2 < q_1$. Thus the equilibrium fees agency i charges are

$$\phi^g(q_i, q_{-i}) = w\left(p^G(q_i, q_{-i})\right) - \delta V(q_2, q_1) \tag{18}$$

and

$$\phi^b(q_i, q_{-i}) = w\left(p^G(q_i, q_{-i})\right) \tag{19}$$

for $i = 1, 2$.

3.6.2 Behavior On the Equilibrium Path

Only agency 1 gives a rating on the equilibrium path by the active agency lemma. We first construct the probability the leading agency is chosen by a bad firm and the probability the leading agency lies. I derive the properties for the value function associated with these behaviors on path.

By claim 16, $x^*(q_1, q_2) = 1 = \beta^*(q_1, q_2)$ for all $q_1 > q_2$ and $q_1 \geq \rho_1^M$. Given the rating fees (18) and (19),

$$V(q_2, q_1) = \frac{(1-\lambda)\delta}{1-\lambda\delta}(1-q_1)V(q_2) \tag{20}$$

$$V(q_1, q_2) = \frac{w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-q_1)}\right) - \lambda\delta V(q_2, q_1)}{1-\lambda\delta}. \tag{21}$$

Thus, $V(.,.)$ is continuous, strictly increasing in the agency's own reputation and strictly decreasing in its competitor's reputation.

For $q_1 < \rho_1^M$, a good rating has zero surplus between the leading agency and a bad firm. I first assume that a bad firm obtains a rating for sure from agency 1 and construct what the leading agency's probability of lying must be in equilibrium under this assumption. Denote by $\tilde{x}^*(q_1, q_2)$ the leading agency's probability of lying under this construction and the value function associated with both agencies by $\left(\tilde{V}(q_1, q_2), \tilde{V}(q_2, q_1)\right)$. For fixed q_2, this construction is similar to that in Mathis et al.(2009).

Claim 18 *The value function $\tilde{V}(.,.)$ thus constructed is continuously increasing in an agency's own reputation and continuously decreasing in its competitor's reputation for all $(q_1, q_2) \in [0,1]^2$.*

Proof. Define $\rho_1^D(q_2) = \rho_1^M$, for all q_2. Define $\tilde{x}^*(q_1, q_2) = 1$ and $\tilde{V}(.,.)$ on $\{(q_1, q_2) : q_1 \geq \rho_1^M\}$ by (20) and (21). Then claim 18 holds on $\{(q_1, q_2) : q_1 \geq \rho_1^M\}$.

Suppose $\rho_k^D(q_2)$ is defined for all q_2, $\rho_k^D(q) \leq \rho_1^M * \rho_{k-1}^D(q)$, and $\tilde{V}(.,.)$ is well-defined such that claim 18 holds on $\{(q_1, q_2) : q_1 \in [\rho_k^D(q_2), 1]\}$. Define $\rho_{k+1}^D(q_2)$ such that

$$w\left(\frac{\lambda}{\lambda + (1-\lambda)\left(1 - \frac{\rho_{k+1}^D}{\rho_k^D(q_2)}\right)}\right) = \delta\tilde{V}\left(\rho_k^D, q_2\right) \quad (22)$$

if $\rho_k^D(q_2) > q_2$ and the solution is no smaller than q_2, and define it as q_2 otherwise. Because the left hand side of (22) is continuously decreasing in ρ_{k+1}^D, and is greater than $\delta V(1) \geq \delta\tilde{V}\left(\rho_k^D, q_2\right)$ when $\rho_{k+1}^D = \rho_1^M * \rho_k^D$ and smaller than $\delta V\left(\rho_k^D, q_2\right)$ when $\rho_{k+1}^D = 0$, there exists a unique solution $\rho_{k+1}^D(q_2) \leq \rho_1^M * \rho_k^D(q_2)$.

Define $\tilde{a}^*\left(\rho_{k+1}^D(q_2), q_2\right) = \frac{\rho_k^D(q_2)}{\rho_{k+1}^D(q_2)}$. For $q_1 \in \left(\rho_{k+1}^D, \rho_k^D\right)$, define the leading agency's probability of lying by (15), where the accuracy $\tilde{a}^*(q_1, q_2)$ is the solution a^* to

$$w\left(\frac{\lambda}{\lambda + (1-\lambda)(1-a^*)}\right) = \delta\tilde{V}\left(\frac{q_1}{a^*}, q_2\right). \quad (23)$$

Claim 19 *$\tilde{a}^*(q_1, q_2)$ and $\chi_1^B(q_1, q_2) = \frac{q_1}{\tilde{a}^*(q_1,q_2)} > \rho_k^D(q_2)$ are both well-defined, continuously increasing in q_1, and continuously decreasing in q_2.*

Proof. See Appendix A.5.1. ■

Because the leading agency's reputation level rises to above ρ_k^D after giving a bad rating, by hypothesis, the property of $\tilde{a}^*(.,.)$, and the fact that $V(q_2, q_1) \leq V(q_2)$ for all $q_2 < q_1$ and strict inequality holds for $q_2 > 0$, claim 18 holds for $\tilde{V}(q_2, q_1)$ on $\{(q_1, q_2) : q_1 \in [\rho_{k+1}^D(q_2), \rho_k^D(q_2))\}$. Because the value of agency 1's good rating is determined by (23), plugging $\beta^*(q_1, q_2) = 1$ into (17), we can immediately see that claim 18 also holds for $\tilde{V}(q_1, q_2)$ on $\{(q_1, q_2) : q_1 \in [\rho_{k+1}^D(q_2), \rho_k^D(q_2))\}$. Because either $\rho_{k+1}^D(q_2) \leq \rho_1^M * \rho_k^D(q_2)$ or $\rho_j^D(q_2) = q_2$ for every $j \geq k+1$, there exists finite $K \in \mathbb{N}$ such that $q_1 \in [\rho_{K+1}^D(q_2), \rho_K^D(q_2)]$ for all $q_1 > q_2$. Claim 18 then follows by induction. ■

Define

$$I = \left\{q_2 : \exists q_1 > q_2 \text{ such that } \tilde{V}(q_1, q_2) \leq \tilde{V}(q_2, q_1)\right\},$$

which is the set of trailing agency reputation levels such that the foregoing construction does not work for some reputation profiles. For all $r \in I$, define

$\phi(r)$ as the unique solution $\phi \geq \sup \left\{ q_1 : \tilde{V}(q_1, q_2) \leq \tilde{V}(q_2, q_1) \right\}$ to

$$\begin{aligned}
& \delta \tilde{V}\left(\frac{\phi}{\tilde{a}^*(\phi, r)}, r\right) \\
= & \frac{\left(1-(1-\lambda)\left(1-\frac{1-a(\phi,r)}{1-a(\phi,r)\frac{r}{\phi}}\right)\delta\right)}{\left(1-(1-\lambda)\left(1-\frac{1-a(\phi,r)}{1-a(\phi,r)\frac{r}{\phi}}\right)\right)} \\
& \times \frac{(1-\lambda)\delta}{1-\lambda\delta-(1-\lambda)\left(1-\frac{1-a(\phi,r)}{1-a(\phi,r)\frac{r}{\phi}}\right)\delta} \\
& \times \left[(1-a(\phi,r))V(r) + \frac{(1-a(\phi,r))a(\phi,r)\frac{r}{\phi}}{1-a(\phi,r)\frac{r}{\phi}}\tilde{V}\left(r, \frac{\phi}{a(\phi,r)}\right)\right]. \quad (24)
\end{aligned}$$

It is well-defined and continuous for all $r \in I$ (claim 35).

Define

$$\begin{aligned}
A &= \{(q_1, q_2) : q_2 \in I,\, q_1 \in (q_2, \phi(q_2))\} \\
C &= \left\{(q_1, q_2) : (q_1, q_2) \in [0,1]^2,\, q_1 > q_2\right\}.
\end{aligned}$$

For $(q_1, q_2) \in C \backslash A$, let the behaviors be described by the previous construction, i.e., $\beta^*(q_1, q_2) = 1$ and $a^*(q_1, q_2) = \tilde{a}^*(q_1, q_2)$. Then, for $q \in C \backslash A$, the values for the leading and trailing agencies are still given by $\tilde{V}(.,.)$ because the leading agency's probability of lying and the future value for both agencies both depend only on the agencies' values at $\chi^B(q_1, q_2) \in C \backslash A$. This further implies that the leading agency is indifferent between lying and not lying because of (23). Moreover, $V(q_1, q_2) > V(q_2, q_1)$ for all $(q_1, q_2) \in C \backslash A$ by construction of ϕ and claim 18.

For $(q_1, q_2) \in A$, let $\beta^*(q_1, q_2) = \frac{1-\tilde{a}^*(\phi(q_2), q_2)}{1-\tilde{a}^*(\phi(q_2), q_2)\frac{q_1}{\phi(q_2)}}$ and $a^*(q_1, q_2) = 1 - \frac{1-\tilde{a}^*(\phi(q_2), q_2)}{\beta^*(q_1, q_2)}$. Then, $x^*(q_1, q_2)$ is well-defined by equation (15) because $a^*(q_1, q_2)$ so defined is in $[q_1, a(\phi(q_2), q_2)]$. Because $\frac{q_1}{a^*(q_1, q_2)} = \phi(q_2)$, the future reputation profiles that will be realized if the players follow this strategy profile are in $C \backslash A$, and thus the values already defined. Thus the equilibrium payoff at (q_1, q_2) is well-defined by (16) and (17). Given the equilibrium probability of lying $x^*(q_1, q_2)$, by the construction of β^* and a^*, agency 1 is indifferent between lying and not lying.

Lastly, when the two agencies have equal reputation level $r \in (0,1)$, define $\beta^*(r, r) = \frac{1}{2}\lim_{q_1 \to r^+} \beta^*(q_1, r)$ and $x^*(r, r) = \lim_{q_1 \to r^+} x^*(q_1, r)$. In Appendix A.5.1, I show that an agency approached by a bad firm is indifferent between lying and not lying in equilibrium.

3.6.3 Properties of the Value Function

We will now see that the payoff associated with the constructed equilibrium is continuous on C and on the section of the diagonal line $\{(r, r) : r \in I\}$, and an increase in the leading agency's reputation increases the payoff for the leading agency, but decreases that for the trailing agency. In addition, the leading agency always has a higher payoff than the trailing agency. Omitted proofs are in the Appendix.

We first observe that the behaviors on the equilibrium path after the equilibrium fee profile is proposed are all continuous on C. This follows immediately from the continuity of $\tilde{a}^*(.,.)$ and $\tilde{V}(.,.)$ and the construction.

Observation $a^*(q_1,q_2)$, $x^*(q_1,q_2)$, $\beta^*(q_1,q_2)$, $V(q_1,q_2)$ and $V(q_2,q_1)$ are all continuous on C.

In addition, the better the leading agency's reputation is, the higher its own payoff is, and the lower the trailing agency's payoff is.

Lemma 20 *$\frac{\partial V}{\partial q_1}(q_1,q_2) > 0$, but $\frac{\partial V}{\partial q_2}(q_1,q_2) < 0$ on C.*

By the construction of $\phi(r)$ for all $r \in I$, when a bad firm obtains no rating with positive probability in equilibrium, the values for the leading agency and trailing agency converge as the former's reputation approaches that of latter, that is, $\lim_{q_1 \to r^+} V(q_1, r) = \lim_{q_1 \to r^+} V(r, q_1)$. Lemma 21 then follows immediately by the continuity of the value function $V(.,.)$ on C. This further implies that an agency's value function is continuous at $q=(r,r)$ for $r \in I$.

Lemma 21 *For $r \in I$, $\lim_{q_1 \to r^+} V(q_1,r) = \lim_{q_2 \to r^-} V(q_2,r) = V(r,r)$.*

Using lemma 21 and lemma 20, we can see that the leading agency always has a higher payoff than the trailing agency.

Lemma 22 *$V(q_1,q_2) > V(q_2,q_1)$ for all $q_1 > q_2$.*

Proof.

$$\begin{aligned} & V(q_1,q_2) - V(q_2,q_1) \\ = \; & V(q_1,q_2) - \lim_{r \to q_2^+} V(r,q_2) - \left(V(q_2,q_1) - \lim_{r \to q_2^+} V(q_2,r) \right) > 0 \end{aligned}$$

because $V(r,q_2)$ is increasing in r and $V(q_2,r)$ is decreasing in r. ■

4 Discussion of assumptions

Fees Are Not Paid if Securities Are Not Issued This assumption is made by both Mathis et al.(2009) and Bolton et al.(forthcoming), and is supposed to capture the conflict of interest in the issuer-pays model and the pressure on rating agencies to issue a good rating. The U.S. Securities and Exchange Commission found that rating agencies often allow "key participants in the rating process to participate in fee discussions." The rating fee that a bad rating forfeits can also be thought of as including future rating business or consulting services. In an article about a reform in the rating fee structure, Reuters cites New York Attorney General Andrew Cuomo as saying that "under the old fee system, the agencies had a financial incentive to assign high ratings because they only received fees if a deal was completed."[3] Levin and Coburn (2010) document a considerable body of email correspondence within Moody's and S&P about the threat of losing deals by not giving an investment bank the desired rating on its securities or by using tougher standards.

A Rating Agency Sets Its Own Rating Fee. This assumption is also made by Bolton et al.(forthcoming). They cited White's (2002) comment that agencies may negotiate fees with regular customers. In S&P's ratings schedule for 2009, the fees for structured finance "range up to 12 basis points," and "higher fees applied to more complex transactions." It also states that fees can be negotiated for "volume issuers and other entities who want multi-year ratings services agreements." Levin and Coburn (2011) document email correspondence about one deal linking a rating with an unusually hefty fee. They also cite a Moody's email from 2007 raising concern over its market share in CDOs and asking if Fitch is cheaper. This fee can also be thought of simply as a way for the agency to choose how much surplus to give to the firm it rates. It can capture the speed of work or the amount of resources and cooperation requested of the issuer. Levin and Coburn (2011) provide incidences suggesting a shift in the relationship between Moody's and the issuers — from the agency being considered a necessary evil to it having to exert great effort just to please the issuers.

Agencies Observe The quality of the Product Before Deciding on Its Rating Fee. It is a simplifying assumption reflecting the fact that an agency has a lot of information about the product at the time rating fees are negotiated. According to a statement by Stephen W. Joynt, the CEO of Fitch, "an objective opinion about the creditworthiness of an issuer can be formed based solely on public information in many jurisdictions."[4] In addition, a rating agency often follows an important firm even if it is not selected to rate its issuances. Therefore, a rating agency knows a good deal about a product before it makes an offer. Moreover, Joynt admits that "structured finance analysts may be involved in fee discussions,"[5] and analysts may learn even more about the product during negotiation. SEC (2008) and Levin and Coburn (2011) both document the presence of analysts when fees are negotiated. Levin and Coburn (2011) cited an email correspondence from Moody's to Merril lynch about an unusually high fee that Moody's is proposing for a deal : "...the rating process

[3] <http://www.reuters.com/article/ousiv/idUSN0528456020080606>

[4] The full quote "Although structured finance analysts may be involved in fee discussions, they are typically senior analysts who understand the need to manage the potential conflict of interest." It is in an email from Fitch to SEC. See <http://www.sec.gov/news/extra/credrate/fitchratings1.htm>

[5] See Note 4.

so far has already shown that the analysis for this deal is far more involved and will continue to be so. We have spent significant amount of resource[s] on this deal and it will be difficult for us to continue with this process if we do not have an agreement on the fee issue." It suggests that by the time fees are negotiated, the agency already has a lot of information about the product.

A Seller Obtains One or No Rating According to Joynt (2002), in structured finance, Fitch is "frequently one of two rating agencies rating a security chosen by the issuer from among the three agencies." The market behavior seemed to change dramatically after Fitch became an important third player.[6] For example, Levin and Coburn (2011) cite a 2006 email in which UBS threatened to do "moodyfitch only cdos" if S&P decided to "use a more conservative rating model." The significant effect of increasing the number of rating agencies from two to three may be driven by regulatory requirements that some products obtain at least two ratings. As a simplified model to capture the effect when issuers have one more degree of freedom in their choice of raters, I assume that a seller obtains at most one rating and compare equilibria when the number of agencies increases from one to two.

Agency with Higher Reputation Has Higher Value The crucial observation in this paper, the zero surplus lemma, does not depend on whether one agency's value is higher than or equal to the other's. Theorem 9 still holds if, instead of the value inequality condition, the market beliefs about the two agencies are disciplined via the beliefs about the average quality of the firms they rate.

Two weaker assumptions provide similar results. The first alternative is to strengthen the *maximum coverage* condition by requiring the following. If agency i strictly prefers rating a bad firm at its equilibrium fee to letting its competitor or no agency rate this firm, then investors' beliefs about the average quality of firms rated by agency i cannot be better than the prior λ, i.e., $\lambda(q_i, q_j) \leq \lambda$.

The second alternative is to require that investors believe the average quality of firms rated by a more reputable agency to be at least as high as the average quality of firms rated by less reputable agency, i.e., $\lambda(q_1, q_2) \geq \lambda(q_2, q_1)$ if $q_1 \geq q_2$.

All proofs pretty much go through except that an active agency is not necessarily the leading agency.

Without any of these conditions to discipline investors' beliefs, an equilibrium under duopoly may exhibit a good rating with higher value than that under monopoly at one reputation profile, at the expense of having a good rating with a much lower value than that under monopoly at a higher reputation profile. In such an equilibrium, it is necessary that when the leading agency has very good reputation, both agencies are active, and the investors believe that the average quality of firms rated by the trailing agency is much better than the

[6] See Becker et al. (2011).

prior and much better than that of firms rated by the leading agency. Therefore, an agency can earn a more favorable market belief about the firms it rates by improving the reputation of its competitor so that it lags more significantly behind its competitor in reputation terms. Hence, an agency may maintain a higher value under duopoly than under monopoly at some reputation profiles by waiting for its competitor to gain a better reputation. Whether such an equilibrium exists depends considerably on the parameters and functional forms. In addition, it requires counterintuitive beliefs to "redistribute" value across reputation profiles.

5 Conclusion

In both the issuer-pays model used in the credit rating industry and the commission model for a sales intermediary, the agency or information intermediary has a short-run incentive to lie about the true quality of the product it rates because of the conflict of interest inherent in the pay structure. The agency gets paid a fee in the current period only if it gives a good rating. It is only the agency's concern for the future value of reputation that may stop it from lying. This paper builds on Mathis et al.(2009) and employs a fully dynamic model that endogenizes the value of reputation to examine the effect of competition on the reputation mechanism. To focus on the business stealing effect, I abstract away many other aspects related to having more information intermediaries, for example, the phenomenon of ratings shopping, the market coverage, or the improved information from having two imperfect signals about quality assuming that the agencies are truthful.

I find that whenever a monopolist might sometimes lie, adding a competitor of equal or poorer reputation can only increase the probability that the agency lies, thereby increasing the probability that a bad firm is invested in, and lowering the value of a good rating issued in this market. It suggests that the rise of Fitch may have caused S&P and Moody's to become more lax by presenting the issuers with a degree of freedom in obtaining two ratings. This is the case because, in general, competition does not reduce the benefits of lying because it does not cause the agency to lower the rating fee for a bad firm. However, competition does lower the agency's cost of lying by forcing it to leave more surplus to a good firm, thereby lowering the value of business at higher reputation levels. It follows that competition causes an agency to lie more often in equilibrium. In other words, because good firms represent better deals and a higher surplus from trade, competition slashes the fee received from a good firm much more than the fee from a bad firm. But the reputation mechanism kicks in exactly when an agency is faced with a bad firm. Therefore, competition increases the incentives to lie.

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A Appendix For Online Publication

A.1 Monopoly Section

A.1.1 Zero Value to Zero Reputation

Lemma 23 $V(0) = 0$.

Proof. We show that $w(p^G) = 0$. Because reputation does not change once the agency is known to be opportunistic, there are zero costs for the agency to give a good rating to any firm. Suppose $w(p^G) > 0$. Then in equilibrium, the agency charges $\phi^v = w(p^G)$ to both $v = b, g$ and both types must obtain a rating with probability 1. In addition, the agency gives a good rating with probability 1 regardless of firm type. But then $p^G = \lambda$ and $w(p^G) = 0$, a contradiction. But then $V(0) = 0 + \delta V(0)$, so $V(0) = 0$. ■

A.1.2 A Bad Rating Has Zero Value

I next show that, under both market structures, a bad rating from agency i has zero value if this agency loses all future business when it becomes known to be opportunistic. It follows from lemma 23 that $w(p^B) = 0$ under monopoly.

Lemma 24 *In any active equilibrium,* $w(p^{B_i}) = 0$ *if* $V(0, q_{-i}) = 0$.

Proof. Suppose to the contrary that $w(p^{B_i}) > 0$. Then securities will be issued and the project quality will be found out after agency i gives a good rating. When the firm is good, agency i loses its reputation by giving a bad rating, but if it gives a good rating, its reputation will remain positive. Because having positive reputation has nonzero value, the cost of good rating for a good firm is negative:

$$\begin{aligned} c_i^g &= \delta V\left(\chi^{B_i g}\right) - \delta V\left(\chi^{G_i g}\right) \\ &= \delta V(0, q_{-i}) - \delta V\left(\chi_i^{G_i g}, q_{-i}\right) \\ &= -\delta V\left(\chi_i^{G_i g}, q_{-i}\right) \\ &< 0. \end{aligned}$$

Suppose investment takes place if i publishes a good rating. Then agency i is paid the fee he sets regardless of the rating he publishes. Thus it is optimal for i to publish a good rating for a good project because this gives rise to positive future reputation, and thus positive future value, while publishing a bad rating will make i lose all reputation, which has zero value given that the opponent has zero reputation too. Contradiction. If investment does not take place after agency i publishes a good rating, then agency i is paid if and only if a bad rating is published. Then when the firm is bad, publishing a bad rating gives i higher future reputation than publishing a good rating. Thus i strictly prefers publishing a bad rating when the firm is bad. Because $p^{B_i} > \lambda$, the probability that a firm that approaches agency i is good must be positive. Thus $p^{G_i} = 1$, and thus securities are issued if the firm obtains a good rating from i. Contradiction. ■

A.1.3 A Good Rating Has Positive Value

Lemma 25 *$w\left(p^G(q)\right) > 0$ for all $q > 0$.*

Proof. Suppose, to the contrary, that securities are never issued. The agency's future reputation thus remains q, and its payoff is

$$V(q) = 0 + \delta V(q).$$

Thus $V(q) = 0$, contradicting the condition that $V(q) > 0$. ■

A.1.4 Zero Outside Option

We obtain that $w\left(p^{\emptyset}\right) = 0$ under both market structures from the following lemma and the maximal coverage condition that $w\left(p^{\emptyset}\right) = 0$ if $\emptyset$ is off the equilibrium path.

Lemma 26 *In an active equilibrium under either monopoly or duopoly, either a good firm obtains a rating with probability 1, or $\emptyset$ is off equilibrium path.*

Proof. We first show that a good firm must approach an agency with positive probability. If a good firm never approaches any agency, then either no firm approaches any agency, and thus the agencies' reputation will not change, or only a bad firm may approach an agency, and thus ratings are of no value because only bad firms obtain any ratings, therefore agencies always give a bad rating to a bad firm and hence reputations do not change in equilibrium. In both cases, agency i obtains no positive fee in the current period and maintain the same reputation and thus same value, i.e.

$$V_i(q) = 0 + V_i(q).$$

Thus $V_i(q) = 0$, contradiction to the assumption of an active equilibrium. Thus we can assume w.l.o.g. that agency 1 is approached by a good firm with positive probability.

Then $w\left(p^{G_1}\right) > w\left(p^{\emptyset}\right)$. Otherwise, a bad firm will never approach agency 1 because it obtains a bad rating from agency 1 with probability at least $q_1 > 0$, and thus even if agency 1 charges zero fee, a bad firm still gets a higher payoff from not obtaining a rating from agency 1. But then $p^{G_1} = 1 > p^{\emptyset}$, contradiction to the hypothesis that $w\left(p^{G_1}\right) \leq w\left(p^{\emptyset}\right)$ because w is strictly increasing for $p \geq \lambda$.

Assume to the contrary that $w\left(p^{\emptyset}\right) > 0$. Because an honest agency always gives a good rating to a good project, and both types of agencies charge the same price, $\chi_1^{G_1 g} \geq q_1 = \chi_1^{\emptyset}$. Suppose a good firm approaches agency 1 and approaches no agency both with positive probability. Then $\phi_1^g = w\left(p^{G_1}\right) - \frac{w\left(p^{\emptyset}\right)}{q+(1-q)y}$ so that a good firm is indifferent between agency 1 and obtaining no rating. Then a bad firm must also randomize between obtaining no rating and approaching some agency i. Otherwise, either $p^{\emptyset} = 1$, a contradiction, or $p^{\emptyset} \leq \lambda$, contradicting the hypothesis that $w\left(p^{\emptyset}\right) > 0$. If a bad firm never approaches agency 1 in equilibrium, then the probability that a firm getting no rating or a rating from agency 2 has a good project is less than λ. Thus either $p^{\emptyset} < \lambda$, a contradiction, or $p^{G_2} < \lambda$, in which case $w\left(p^{G_2}\right) = 0$ and thus approaching agency 2 is strictly dominated by obtaining no rating, a contradiction. So a bad firm must also put positive probability on obtaining no rating and also approaching agency 1. For a bad firm to be indifferent between obtaining no rating and approaching agency 1,

$$\phi_1^b = w\left(p^{G_1}\right) - \frac{w\left(p^{\emptyset}\right)}{(1-q)\,x_1^*}.$$

Thus agency 1 must give a good rating to a bad firm with positive probability in equilibrium. Thus

$$\begin{aligned} \phi_1^b &\geq V_1\left(\chi^{B_1}\right) - V_1\left(\chi^{G_1 b}\right) \\ &= V_1\left(\chi^{B_1}\right). \end{aligned}$$

I will show that $\chi_1^{G_1 g} = q_1$. Suppose otherwise, then it must be the case that $y_1 \in (0,1)$. Thus

$$\begin{aligned} w\left(p^{G_1}\right) - \frac{w\left(p^{\emptyset}\right)}{q+(1-q)\,y_1^*} &= \phi_1^g \\ &= V_1\left(\chi^{B_1}\right) - V_1\left(\chi^{G_1 g}\right) \\ &< V_1\left(\chi^{B_1}\right) \\ &\leq w\left(p^{G_1}\right) - \frac{w\left(p^{\emptyset}\right)}{(1-q)\,x_1^*}. \end{aligned}$$

Thus $q+(1-q)\,y_1^* < (1-q)\,x_1^*$. Let λ_1 denote the probability that a firm rated by agency 1 is good. Then

$$\begin{aligned} p^{G_1} &= \frac{\lambda_1\left(q+(1-q)\,y_1^*\right)}{\lambda_1\left(q+(1-q)\,y_1^*\right)+(1-\lambda_1)\,(1-q)\,x} \\ &= \frac{\lambda_1}{\lambda_1+(1-\lambda_1)\,\frac{(1-q)x}{q+(1-q)y_1^*}} \\ &< \frac{\lambda_1}{\lambda_1+(1-\lambda_1)\,\frac{1-(1-q)x_1^*}{\left(1-\left(q+(1-q)y_1^*\right)\right)}} \\ &= p^{B_1}. \end{aligned}$$

But $w\left(p^{B_1}\right)=0$. Thus $w\left(p^{G_1}\right)=0$, contradiction. Likewise, if a good firm approaches agency 2 (under the duopoly model) with positive probability, then the same argument shows that $\chi_2^{G_2 g}=q_2$ and $y_2^*=1$. Thus, whether or not the good firm approaches agency 1, the reputation profile does not change, and hence agency 1's future payoffs are the same. But agency 1 obtains $\phi_1^g = w\left(p^{G_1}\right)-w\left(p^{\emptyset}\right)>0$ if the good firm approaches agency 1. Then as long as $w\left(p^{G_1}\right)-w\left(p^{\emptyset}\right)>\delta V_1\left(\chi^{B_1}\right)-\delta V_1\left(\chi^{G_1 g}\right)$, after charging $\phi_1^g-\varepsilon$ for $\varepsilon>0$ sufficiently small, agency 1 will still give a good rating to the good firm and thus the good firm will strictly prefer approaching agency 1 and will choose agency 1 with probability 1. Therefore, agency 1 can increase its payoff by charging $\phi_1^g-\varepsilon$, a contradiction to ϕ_1^g being his equilibrium fee. Thus it must be the case that $w\left(p^{G_1}\right)-w\left(p^{\emptyset}\right)=\delta V_1\left(\chi^{B_1}\right)-\delta V_1\left(\chi^{G_1 g}\right)$, so by charging a lower rating fee, agency 1 will subsequently issue a bad rating for sure and thus lose all business from the good firm. But then

$$w\left(p^{G_1}\right)-\frac{w\left(p^{\emptyset}\right)}{(1-q_1)\,x_1^*} < w\left(p^{G_1}\right)-w\left(p^{\emptyset}\right) < \delta V_1\left(\chi^{B_1}\right).$$

We have shown that in equilibrium, a bad firm must approach agency 1 with positive probability, thus $\phi_1^b \le w\left(p^{G_1}\right)-\frac{w\left(p^{\emptyset}\right)}{(1-q_1)x_1^*}$, so that a bad firm weakly prefers approaching agency 1 to obtaining no rating. But then the fee is lower than agency 1's reputation cost. Thus $x_1^*=0$. But then a bad firm strictly prefers obtaining no rating to approaching agency 1, a contradiction. ■

A.1.5 Proof for Lemma 4

The complication of the proof comes from not requiring the value function to be increasing in reputation.

Proof for Lemma 4. Suppose to the contrary that $w\left(p^{G}\right)=c^g$. Then, $w\left(p^{G}\right)<c^b$, and thus the opportunistic monopolist gives a bad firm a bad rating for sure at any fee that a rational bad firm may accept. It follows that

$p^G = 1$ and

$$\begin{aligned}\chi^B &= \frac{(1-\lambda) q}{(1-\lambda) q + ((1-\lambda)(1-q) + \lambda (1-q)(1-y^*))} \\ &\leq q \\ &< \frac{q}{q + (1-q) y^*} \\ &= \chi^{Gg}.\end{aligned}$$

Thus, $w(1) = \delta V(\chi^B) - \delta V(\chi^{Gg})$. It is a contradiction if $V(.)$ is weakly increasing.

However, we can see the contradiction even without requiring the equilibrium value function increasing.

Because $\delta V(q') \leq \delta \max\left\{\frac{\lambda w(1)}{1-\delta}, \frac{w(1)}{1-\lambda\delta}\right\}$ for all $q' \in [0,1]$, it follows that $\frac{\delta}{1-\lambda\delta} > 1$. Because $w(p^G(q)) = c^g$,

$$\begin{aligned}V(q) &= \lambda\left(w(1) + \delta V(\chi^{Gg})\right) + (1-\lambda)\delta V(\chi^B) \\ &= \delta V(\chi^B) + \lambda\delta V(\chi^{Gg}) \\ &\geq w(1) + \lambda\delta V(\chi^{Gg}).\end{aligned}$$

Because $V(\chi^{Gg}) > 0$, $V(q) < \delta V(\chi^B)$. If $V(q) \leq V(\chi^{Gg})$, then $V(q) \geq \frac{\delta V(\chi^B)}{1-\lambda\delta} > V(\chi^B)$ because $\frac{\delta}{1-\lambda\delta} > 1$, contradicting the fact that $V(\chi^{Gg}) < \delta V(\chi^B)$. So $V(q) > V(\chi^{Gg})$ and $\chi^{Gg} > q$.

Because $V(q) < V(\chi^B) \leq \frac{\lambda}{1-\delta} w(1)$, which is the payoff if the agency is believed never to lie at any reputation level on the equilibrium path, it must be the case that there exists $q' \in (0,1)$ such that agency the lies with positive probability. Because $c^g < c^b$, if $x^*(q') > 0$, then $y^*(q') = 1$, and thus $\chi^{Gg}(q') = q'$ and $\chi^B(q') > q'$. If $x^*(q') = 1$, then $w(p^G) < w(1) < \delta V(1) = \delta V(\chi^B)$, contradicting the hypothesis that $x^*(q') > 0$. Thus, $x^*(q') \in (0,1)$, and it follows that $V(q') = \frac{\delta V(\chi^B(q'))}{1-\lambda\delta}$. Then, $V(q') > V(\chi^B(q'))$. Define $q_0 = q'$, and $q_1 = \chi^B(q')$. Then, we have $q_1 > q_0$ and

$$V(q_{k+1}) = \left(\frac{\delta}{1-\lambda\delta}\right)^{-1} V(q_k) < V(q_k) \tag{25}$$

for $k = 0$.

Suppose the agency does not lie at q_1. If $x^*(q_1) = 0$ and $y^*(q_1) = 1$, then $\chi^B(q_1) = q_1$. It follows that $V(q_1) = \frac{\lambda w(1)}{1-\delta} \geq V(q_0)$, a contradiction of (25). If $y^* < 1$, then $\chi^{Gg}(q_1) > q_0$, and $V(\chi^{Gg}(q_1)) < V(q_1) = \left(\frac{\delta}{1-\lambda\delta}\right)^{-1} V(q_0)$. Define $q_2 = \chi^{Gg}(q_1)$. Because $V(\chi^B(q_1)) > V(q_1)$,

$$V(q_k) \geq \lambda w(1) + \lambda\delta V(q_{k+1}) + (1-\lambda)\delta V(q_k) \tag{26}$$

$$V(q_k) > V(q_{k+1}) \tag{27}$$

for $k = 1$.

Suppose the agency lies with positive probability at q_1. Then using previous arguments, there exists $q_2 > q_1$ such that $V(q_2) = \left(\frac{\delta}{1-\lambda\delta}\right)^{-1} V(q_1)$. By induction, there exists a strictly increasing sequence $\{q_k\}_{k=0}^{\infty}$ such that either (25) holds or (26) and (27) hold for each k. Because $\{q_k\}_k$ is strictly increasing and $\{V(q_k)\}_k$ is strictly decreasing, these limits exist. If (26) and (27) hold for infinitely many k, then

$$\lim_{k\to\infty} V(q_k) \geq \frac{\lambda w(1)}{1-\delta}.$$

But, then $V(q_k) > \frac{\lambda w(1)}{1-\delta}$ because $V(q_k)$ is strictly decreasing, a contradiction. If (25) holds for infinitely many k, then $\lim_{k\to\infty} V(q_k) = 0$ by (25). Because $V(q_k) \geq w\left(p^G(q_k)\right)$, $\lim_{k\to\infty} w\left(p^G(q_k)\right) = 0$. Thus $a_k^* \to \underline{a} < 1$ where $\underline{a} := \sup\left\{a : w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-a)}\right) = 0\right\}$. Then $\lim_{k\to\infty} q_{k+1} = \lim_{k\to\infty} \chi^B(q_k) = \lim_{k\to\infty} \frac{q_k}{a_k^*} = \frac{\lim_{k\to\infty} q_k}{\underline{a}} > \lim_{k\to\infty} q_k$, a contradiction.

Therefore, there must be surplus from trade with a good firm. Then trade must occur with probability 1. ■

A.2 Proof for the Preliminary Section under Duopoly

In this section we prove lemma 7.

The main reason that $V(0, q_j) = 0$ is that once an agency is known to be opportunistic, its reputation does not change. The complication in the proof arises because an agency's value may change with its competitor's reputation even though its own reputation remains the same.

Once we see that $V(0, q_j) = 0$ for all q_j, then it is easy to show that $w\left(p^{B_i}(q)\right) = 0$ using the same type of argument as that in the monopoly case because the competitor's reputation does not depend on whether agency i gives a good or bad rating.

It then follows from lemma 26 and the *maximum coverage* condition that $w\left(p^{\emptyset}\right) = 0$.

Lemma 27 $V(0,0) = 0$ *under duopoly.*

Proof. We first see that if agency i does not rate a firm at a positive fee at all, then $V(0,0) = 0 + \delta V(0,0)$, so $V(0,0) = 0$. But in a symmetric Markov Perfect equilibrium, the maximal surplus the two agencies can leave a firm of type v must be the same because they have equal reputations. Thus competition must drive rating fee down to an agency's cost to provide a good rating, which is 0. It then follows that $V(0,0) = 0$. ■

Lemma 28 $V_1(0,1) = 0$.

Proof. Reputation profile does not change regardless of firm type or rating. Suppose $\max\{w(p^{m_1}) : m_1 = G_1, B_1\} > 0$. Assume w.l.o.g. that $w\left(p^{G_1}\right) = \min\{w(p^{m_1}) > 0 : m_i = G_i, B_i\}$. Then a bad firm strictly prefers to approach

1 at any fee $\phi_1^b < w\left(p^{G_1}\right)$. Thus in equilibrium, a bad firm approaches 1 with probability 1. Thus $\lambda_1 \le \lambda$. Thus it cannot be the case that $w\left(p^{G_1}\right) w\left(p^{B_1}\right) > 0$. It follows that $w\left(p^{B_1}\right) = 0$ and securities won't be issued if agency 1 issues a bad rating. But then agency 1 will publish G_1 for a bad firm with probability 1, because cost is 0 and 1 gets $w\left(p^{G_1}\right) > 0$ in the current period iff 1 publishes a good rating. Thus $p^{G_1} \le \lambda$ and $w\left(p^{G_1}\right) = 0$, contradiction. Thus $\max\left\{w\left(p^{m_i}\right) : m_i = G_i, B_i\right\} = 0$ and 1 does not get paid in the current period. Thus $V(0,1) = 0 + \delta V(0,1)$. So $V(0,1) = 0$. ■

Lemma 29 *$V(0, q_{-i}) = 0$ for all $q_{-i} \in [0,1]$.*

Proof. It suffices to consider $q_{-i} \in (0,1)$. Suppose to the contrary that $V(0, q_{-i}) > 0$. If $-i$ does not publish a rating for any firm in equilibrium, then the reputation profile does not change. Then the same rationale for proving that $V_1(0,1) = 0$ can be used to show that $V(0, q_{-i}) = 0$, contradiction. Thus $-i$ must publish a rating for a firm with positive probability.

Because $q_{-i} > 0$, $\chi_{-i}^{G_{-i}g} > 0$. Thus

$$\begin{aligned}
c_{-i}^g &= V_{-i}\left(\chi^{B_{-i}g}\right) - V_{-i}\left(\chi^{G_{-i}g}\right) \\
&= V_{-i}\left(\chi^{B_{-i}g}\right) - V_{-i}\left(\chi_{-i}^{G_{-i}g}, 0\right) \\
&< V_{-i}\left(\chi^{B_{-i}g}\right) - 0 \\
&= V_{-i}\left(\chi^{B_{-i}g}\right) - V_{-i}(0,0) \\
&\le V_{-i}\left(\chi^{B_{-i}b}\right) - V_{-i}(0,0) \\
&= c_{-i}^b.
\end{aligned}$$

Therefore the cost for $-i$ to publish a good rating is strictly higher when the firm is bad than when the firm is good. [Once $-i$ lies, value becomes $V(0,0) = 0$.] Thus, $w\left(p^{B_{-i}}\right) = 0$. Suppose that $w\left(p^{G_{-i}}\right) - w\left(p^{\emptyset}\right) < c_{-i}^g$. Then the cost for $-i$ to give a good rating is higher than the maximum rating fee at which a firm is willing to accept, and thus an opportunistic agency $-i$ will give a good rating with probability 0. But then a good rating must come from an honest agency $-i$, and is given only to a good firm, and a bad rating may come from an opportunistic agency $-i$. Thus $p^{G_{-i}} = 1$ and $\chi_{-i}^{G_{-i}g} = 1 > \chi_{-i}^{B_{-i}}$. Thus $c_{-i}^g \le 0 \le w(1) - w\left(p^{\emptyset}\right) = w\left(p^{G_1}\right) - w\left(p^{\emptyset}\right)$, contradiction to the hypothesis. Then $w\left(p^{G_{-i}}\right) - w\left(p^{\emptyset}\right) \ge c_{-i}^g$.

Claim 30 $w\left(p^{G_{-i}}\right) - w\left(p^{\emptyset}\right) > c_{-i}^g$

Proof. We have shown that $w\left(p^{G_{-i}}\right) - w\left(p^{\emptyset}\right) \ge c_{-i}^g$. Suppose equality holds. Then $w\left(p^{G_{-i}}\right) - w\left(p^{\emptyset}\right) < c_{-i}^b$. Because a firm does not approach $-i$ at any rating fee $\phi_{-i} > w\left(p^{G_{-i}}\right)$, either no bad firm approaches $-i$, or $-i$ gives always gives a bad rating to a bad firm, i.e. $x_{-i}^* = 0$. Thus either B_{-i} is off the equilibrium path, or $\chi_{-i}^{B_{-i}} \le q_{-i}$, and equality holds iff $y_{-i}^* = 1$ or good firm never approaches $-i$. If B_{-i} is off equilibrium path, then $-i$ must rate a good firm with positive probability and gives a good firm a good rating with probability 1, in which case $\chi_{-i}^{G_{-i}g} = q_{-i}$. If $\chi_{-i}^{B_{-i}} = q_{-i}$, then either $y_{-i}^{*} = 1$ and thus $\chi_{-i}^{G_{-i}g} = q_{-i}$ or $y_{-i}^{*} < 1$ but a good firm does not approach $-i$. If B_{-i} is off path of if $y_{-i}^{*} = 1$, $-i$'s reputation does not change in equilibrium. If a good firm does not approach $-i$, then G_{-i} is off path and $\chi_{-i}^{G_{-i}g} = q_{-i}$ and $-i$'s reputation does not change on path either. By previous arguments, $V_i(0, q_{-i}) = 0$, contradiction. Thus it must be the case that $\chi_{-i}^{B_{-i}} < q_{-i}$ and a good firm approaches $-i$ with positive probability.

Thus $y_{-i}^{*} < 1$ and $w\left(p^{G_{-i}}\right) = w(1)$. Then $\chi_{-i}^{G_{-i}g} > q_{-i} > \chi_{-i}^{B_{-i}}$ and $w(1) - w\left(p^{\emptyset}\right) = \delta V_{-i}\left(0, \chi_{-i}^{B_{-i}}\right) - \delta V_{-i}\left(0, \chi_{-i}^{G_{-i}g}\right)$. Then $w\left(p^{\emptyset}\right) = 0$ because otherwise, a good firm strictly prefers getting no rating to paying $w(1) - w\left(p^{\emptyset}\right)$ for a good rating with probability $y_{-i}^{*} < 1$. But then

$$\begin{aligned} w(1) &= \delta V_{-i}\left(0, \chi_{-i}^{B_{-i}}\right) - \delta V_{-i}\left(0, \chi_{-i}^{G_{-i}g}\right) \\ &< \delta V_{-i}\left(0, \chi_{-i}^{B_{-i}}\right) \\ &\leq \max\left\{\frac{\delta}{1-\lambda\delta} w(1), \delta\frac{\lambda}{1-\delta} w(1)\right\}, \end{aligned}$$

i.e.

$$1 \leq \frac{\delta}{1-\lambda\delta}.$$

We thus obtain a contradiction because $\frac{\delta}{1-\lambda\delta} < 1$. ■

Claim 31 $w\left(p^{B_{-i}}\right) = 0$.

Proof. If agency $-i$ loses its reputation, then its value is $V(0,0) = 0$. Therefore, cost of a good rating is higher for a bad firm. The same rationale for the monopoly case applies. ■

Claim 32 *Reputation of agency $-i$ remains q_{-i} if the firm is good.*

Proof. If agency $-i$ rates a good firm with zero probability, then $p^{G_{-i}} = 0$ because we have shown that $-i$ must publish a rating with positive probability. Thus $w\left(p^{G_{-i}}\right) - w\left(p^{\emptyset}\right) \leq 0 < \delta V_{-i}\left(0, \chi_{-i}^{B_{-i}}\right) - \delta V(0,0) = c_{-i}^{b}$, $-i$ will never give a good rating to a bad firm at any fee that a bad firm will accept. Thus $\chi_{-i}^{B_{-i}} = q_{-i}$ and the only rating $-i$ publishes on path is B_{-i}. Therefore reputation profiles do not change regardless of firm type and rating, contradiction. Thus $-i$ must rate a good seller with positive probability. If $w\left(p^{B_{-i}}\right) > 0$, Because $w\left(p^{G_{-i}}\right) - w\left(p^{\emptyset}\right) > c_{-i}^{g}$, by the key lemma, if agency $-i$ rates a good firm with positive probability, it must give good rating to a good firm with probability 1. and thus $y_{-i}^{*} = 1$. Therefore, $\chi_{-i}^{G_{-i}g} = q_{-i}$. So conditional on a good firm, reputation profile does not change. ■

Claim 33 $V(0, q_{-i}) = \gamma V\left(0, \chi_{-i}^{B_{-i}}\right)$ *where* $\chi_{-i}^{B_{-i}} > q_{-i}$ *and* $\gamma \in \left(0, 1 - \frac{1-\delta}{1-\lambda\delta}\right)$.

Proof. Let β_{-i} denote the probability a bad firm approaches $-i$. After a bad firm approaches $-i$, either $-i$ lies and both agencies have zero reputation, or $-i$ does not lie and the reputation profile becomes $\left(0, \chi_{-i}^{B_{-i}}\right)$. If a bad firm does not approach $-i$, then reputation profile remains $(0, q_{-i})$ whether or not i rates the bad firm. Thus, if i does not rate a firm with positive probability, or if i does not rate a bad firm at a positive fee with positive probability,

$$\begin{aligned} & V(0, q_{-i}) \\ = \; & \lambda\delta V(0, q_{-i}) \\ & + (1-\lambda)\delta \begin{bmatrix} \left(1-\beta_{-i}\right) V(0, q_{-i}) + \beta_{-i}(1-q_{-i}) x_{-i}^{*} V(0,0) \\ +\beta_{-i}\left(1-(1-q_{-i}) x_{-i}^{*}\right) V\left(0, \chi_{-i}^{B_{-i}}\right) \end{bmatrix}. \end{aligned}$$

Because $V(0,0) = 0$,

$$\begin{aligned} & V(0, q_{-i}) \\ = \; & \frac{(1-\lambda)\delta}{1-\lambda\delta-(1-\lambda)\delta\left(1-\beta_{-i}\right)} \beta_{-i}\left(1-(1-q_{-i}) x_{-i}^{*}\right) V\left(0, \chi_{-i}^{B_{-i}}\right). \end{aligned}$$

We are done because $\frac{(1-\lambda)\delta}{1-\lambda\delta-(1-\lambda)\delta\left(1-\beta_{-i}\right)} \beta_{-i}\left(1-(1-q_{-i}) x_{-i}^{*}\right)$ belongs to $\left(0, 1 - \frac{1-\delta}{1-\lambda\delta}\right)$.

If i rates a bad firm at a positive fee with positive probability, then we have $\max\left\{w\left(p^{G_i}\right), w\left(p^{B_i}\right)\right\} > w\left(p^{\emptyset}\right)$ because otherwise it is not a best response for a bad firm to pay a positive fee for i's rating. Then i must rate a good firm with positive probability. We have shown that $-i$ must rate a good firm with positive probability. Because reputation profiles are the same no matter who rates the good firm, an agency strictly prefers the business of a good firm as long as it gets positive fee. Thus in equilibrium of Bertrand competition, both must charge a fee equal to their cost of providing a good rating, and leave the same surplus to a good firm. Because i has zero cost to provide a good rating, i must charge zero fee to a good firm and

$$y_i^{*} * w\left(p^{G_i}\right) + (1-y_i^{*}) w\left(p^{B_i}\right) = w\left(p^{G_{-i}}\right) - c_{-i}^{g}.$$

If a bad firm never approaches $-i$, or if $x_{-i}^{*} = 0$, reputation profile remain $(0, q_{-i})$ conditional on a bad firm no matter who the bad firm approaches in equilibrium. But then $V(0, q_{-i}) = 0$, contradiction. So it must be that a bad firm approaches $-i$ with positive probability and $x_{-i}^{*} > 0$. By the key lemma, either $w\left(p^{G_{-i}}\right) = c_{-i}^{b}$, or $x_{-i}^{*} = 1$. If $x_{-i}^{*} = 1$, then after a bad firm approaches $-i$, $-i$'s reputation becomes either 1 or 0. Thus i's payoff becomes 0 because $V_i(0,1) = V_i(0,0) = 0$. So i strictly prefers rating a bad firm at any fee because after rating a bad firm i's future payoff is $\delta V_i(0, q_{-i}) > 0$ by hypothesis. Thus it must be that $\phi_i^{b} = 0$ because otherwise, i can lower fee to attract a bad firm. Contradiction to the hypothesis that i rates a bad firm at a positive fee with positive probability.

So $w\left(p^{G_{-i}}\right)-w\left(p^{\emptyset}\right)=c_{-i}^{b}=\delta V_{-i}\left(\chi^{B_{-i}},0\right)$ and $x_{-i}^{*}\in(0,1)$. Then

$$x_{i}^{*}w\left(p^{G_{i}}\right)+\left(1-x_{i}^{*}\right)w\left(p^{B_{i}}\right)-\phi_{i}^{b}=w\left(p^{G_{-i}}\right)-\phi_{-i}^{b}=w\left(p^{\emptyset}\right).$$

Because $x_{i}^{*}w\left(p^{G_{i}}\right)+\left(1-x_{i}^{*}\right)w\left(p^{B_{i}}\right)>0$ and i has zero cost of providing a good rating, i can charge a lower but still positive fee to a bad firm to steal business from agency $-i$. If i rates a bad firm with positive probability which is not 1 in equilibrium, then i must be indifferent between rating a bad firm and not rating a bad firm. That β_i denote the probability that a bad firm approaches i, then

$$\phi_{i}^{b}+\delta V_{i}\left(0,q_{-i}\right)=\delta\left(\begin{array}{c}\frac{1-\beta_{i}-\beta_{-i}}{1-\beta_{i}}V\left(0,q_{-i}\right)+\frac{\beta_{-i}}{1-\beta_{i}}\left(1-q_{-i}\right)x_{-i}^{*}V\left(0,0\right)\\+\frac{\beta_{-i}}{1-\beta_{i}}\left(1-\left(1-q_{-i}\right)x_{-i}^{*}\right)V_{i}\left(0,\chi_{-i}^{B_{-i}}\right)\end{array}\right).$$

Thus

$$\begin{aligned}V_{i}\left(0,q_{-i}\right)&=\lambda\delta V\left(0,q_{-i}\right)\\&\quad+\left(1-\lambda\right)\delta\left(\begin{array}{c}\frac{1-\beta_{i}-\beta_{-i}}{1-\beta_{i}}V\left(0,q_{-i}\right)+\frac{\beta_{-i}}{1-\beta_{i}}\left(1-q_{-i}\right)x_{-i}^{*}V\left(0,0\right)\\+\frac{\beta_{-i}}{1-\beta_{i}}\left(1-\left(1-q_{-i}\right)x_{-i}^{*}\right)V_{i}\left(0,\chi_{-i}^{B_{-i}}\right)\end{array}\right).\end{aligned}$$

Because $V\left(0,0\right)=0$,

$$V\left(0,q_{-i}\right)=\frac{\left(1-\lambda\right)\delta\frac{\beta_{-i}}{1-\beta_{i}}\left(1-\left(1-q_{-i}\right)x_{-i}^{*}\right)}{1-\lambda\delta-\left(1-\lambda\right)\delta\frac{1-\beta_{i}-\beta_{-i}}{1-\beta_{i}}}V_{i}\left(0,\chi_{-i}^{B_{-i}}\right).$$

We are done because $\frac{(1-\lambda)\delta\frac{\beta_{-i}}{1-\beta_{i}}\left(1-(1-q_{-i})x_{-i}^{*}\right)}{1-\lambda\delta-(1-\lambda)\delta\frac{1-\beta_{i}-\beta_{-i}}{1-\beta_{i}}}\in\left(0,1-\frac{1-\delta}{1-\lambda\delta}\right)$. Because $x_{-i}^{*}\in(0,1)$ and $y_{-i}^{*}=1$, $\chi_{-i}^{B_{-i}}>q_{-i}$. ■

Proof. We can now prove lemma 29. Write $\chi_{-i}^{B_{-i}}=q^{1}$. Thus $V\left(0,q_{-i}\right)=\gamma_{1}V\left(0,q^{1}\right)$ for some $\gamma_{1}\in(0,1)$. It follows that $V\left(0,q^{1}\right)>0$. Thus there exists $q^{2}>q^{1}$ such that $V\left(0,q^{1}\right)=\gamma_{2}V\left(0,q^{2}\right)$. By induction, we can construction a strictly increasing sequence $\left\{q^{k}\right\}_{k=0}^{\infty}$ in $[0,1]$ with $q^{0}=q_{-i}$. Because the maximum fee an agency can possibly receive is $w\left(1\right)$, $V\left(q\right)\leq\frac{w(1)}{1-\delta}$. Thus

$$\begin{aligned}V\left(0,q_{-i}\right)&=\left(\prod_{k=1}^{K}\gamma_{k}\right)V\left(0,q^{K}\right)\\&\leq\left(\prod_{k=1}^{K}\gamma_{k}\right)\frac{w\left(1\right)}{1-\delta}\\&<\left(1-\frac{1-\delta}{1-\lambda\delta}\right)^{K}\frac{w\left(1\right)}{1-\delta}\end{aligned}$$

for all $K = 1, 2, ...$ Thus

$$\begin{aligned} V(0, q_{-i}) &\leq \lim_{K\to\infty} \left(1 - \frac{1-\delta}{1-\lambda\delta}\right)^K \frac{w(1)}{1-\delta} \\ &= 0. \end{aligned}$$

Contradiction. Therefore, $V(0, q_{-i}) = 0$. ■ ■

A.3 Proofs For the Active Agency Lemma

Proof for lemma 13.

I finish the rest of the cases.

If $w\left(p^{G_i}\right) > \delta V\left(\chi_i^{B_i}, q_j\right)$ but $w\left(p^{G_j}\right) = \delta V\left(\chi_j^{B_j}, q_i\right)$, then by the zero-surplus lemma (lemma 10), $x_i^* = 1$. Thus, $a_i^* = q_i$ and $\chi_i^{B_i} = 1$. Agency i then prefers to rate a bad firm at any fee because the future value resulting from giving a bad rating is weakly higher than that from any other possible reputation profile, particularly the value after the bad firm obtains no rating or a rating from its competitor. The preference is strict if i charges more than $\delta V\left(\chi_i^{B_i}, q_j\right)$. Therefore, in equilibrium, a bad firm must be rated by agency i with probability 1, because agency i can give a positive surplus to a bad firm, whereas agency j can only give zero surplus. But then a good firm must also be rated by agency i with probability 1, because otherwise, $w\left(p^{G_j}\right) = w(1) > \delta V(1) \geq \delta V\left(\chi_j^{B_j}, q_i\right)$, contradicting the hypothesis. Thus, agency i is the only active agency, and both types of firm obtain i's rating with probability 1.

To show that $i = 1$, I demonstrate that the active agency must have a higher value than the inactive agency. Because agency i can always give a bad rating and get $\delta V(1)$ from the resulting future reputation, $V_i(q) \geq \delta V(1)$. I now establish that agency j must have a lower value. Suppose the firm is bad. If agency i is opportunistic, agency i will lie for sure and lose all of its reputation, and thus agency j becomes a monopolist and earns $V(q_j)$ from the next period onwards. On the other hand, if agency i is honest, agency i will give a bad rating and become believed to be honest, and thus agency j's future value will be zero. When the firm is good, future reputation profile remains unchanged. Thus $V_j(q) = \frac{(1-\lambda)(1-q_i)}{1-\lambda\delta}\delta V(q_j) < \delta V(1) \leq V_i(q)$. Because leading agency has a higher value, $i = 1$.

When $w\left(p^{G_i}\right) > \delta V\left(\chi_i^{B_i}, q_j\right)$ for $i = 1, 2$, then $x_i^* = 1$ by the zero-surplus lemma (lemma 10), and thus $\chi_i^{B_i} = 1$, for $i = 1, 2$. Suppose to the contrary that agency 2 is active. Then it must be the case that $w\left(p^{G_2}\right) > w\left(p^{G_1}\right)$ so that the maximal surplus agency 2 can leave a good firm is at least as big that agency 1 can leave. Because an agency earns perfect reputation after issuing a bad rating, it prefers rating a bad firm to letting its competitor or no agency rate it. Thus, it will also compete for a bad firm's business by driving rating fee down to cost if necessary. Because an honest agency does not lie, the maximal surplus agency i can leave a bad firm is

$$(1-q_i)\left(w\left(p^{G_i}\right)-\delta V(1)\right).$$

If $w\left(p^{G_2}\right)>w\left(p^{G_1}\right)$, then a bad firm must approach agency 2 with probability 1 in equilibrium. If agency 1 is active, then it rates only good firms, and thus $w\left(p^{G_1}\right)=w(1)>w\left(p^{G_2}\right)$, a contradiction. Thus, agency 1 must be inactive. But, using the same argument as in the previous case, the inactive agency must have a lower value than the active agency, another contradiction. ■

A.4 Proofs that the Value of Reputation Goes Down

First I show that value of reputation for the trailing agency is lower under duopoly.

Claim 34 *$V(q_2,q_1)<V(q_2)$ for all $q_1>q_2$ and $(q_1,q_2)\in(0,1)^2$.*

Proof. More specifically, when the firm is bad, with probability $a_1^*=1-(1-q_1)x_1^*$, agency 1 will give it a bad rating, and its reputation will rise to $\chi_1^{B_1}=\frac{q_1}{a_1^*}$; but, with probability $1-a_1^*$, agency 1 will lie, and its reputation will drop to zero. If the firm is good, or if this bad firm does not obtain a rating, reputation profile does not change. Thus,

$$\begin{aligned}V(r,q) &= \frac{(1-\lambda)\beta(q,r)}{1-\lambda\delta-(1-\lambda)\delta(1-\beta(q,r))}\\ &\quad\times\delta\left[a^*(q,r)V\left(r,\frac{q}{a^*(q,r)}\right)+(1-a^*(q,r))V(r)\right]\\ &< a^*(q,r)V\left(r,\frac{q}{a^*(q,r)}\right)+(1-a^*(q,r))V(r).\end{aligned}$$

Define $q_0=q$. Given $q_k<1$, define $q_{k+1}=\frac{q_k}{a^*(q_k,r)}$. Then, for all k, there exists $\left(\alpha_0^k,...\alpha_k^k\right)$ such that $\sum_{j=0}^k\alpha_0^k=1$ and

$$V(r,q)<\left(\sum_{j=0}^{k-1}\alpha_j^k\right)V(r)+\alpha_k^kV(r,q_k).$$

If there exists K such that $q_K=1$, then $V(r,q)<V(r)$. Otherwise, $\lim_{k\to\infty}q_k=\overline{q}<1$ and $\lim_{k\to\infty}a^*(q_k,r)=1$ and $w\left(p^{G_1}(q_k,r)\right)=\delta V(q_{k+1},r)$. Because the leading agency cannot charge more than the value of its good rating,

$$\begin{aligned}V(q_k,r) &\le \frac{(\lambda+(1-\lambda)\beta(q_k,r))w\left(p^{G_1}\right)}{1-\lambda\delta-(1-\lambda)\delta(1-\beta(q_k,r))}\\ &= \frac{(\lambda+(1-\lambda)\beta(q_k,r))\delta V(q_{k+1},r)}{1-\lambda\delta-(1-\lambda)\delta(1-\beta(q_k,r))}\\ &< \left(1-\frac{1-\delta(1+\lambda)}{1-\delta+(1-\lambda)\delta}\right)V(q_{k+1},r).\end{aligned}$$

But then, $V(q_k, r) = \left(1 - \frac{1-\delta(1+\lambda)}{1-\delta+(1-\lambda)\delta}\right)^{-k} V(q, r) \leq V(1)$ for all k, and thus $V(q, r) = 0$ because $\frac{1-\delta(1+\lambda)}{1-\delta+(1-\lambda)\delta} \in (0, 1)$, a contradiction. ■

Next I show that the leading agency lies for sure and a bad firm obtains a rating for sure when $q_1 \geq \rho_1^M$.

Proof for Claim 16. When $q_1 \in [\rho_1^M, 1)$, by definition of ρ_1^M, agency 1's reputation is sufficiently high that even if the opportunistic agency 1 lies for sure, the value of its good rating is still higher than the discounted value of even a perfect reputation $\delta V(1)$. It is even higher if a bad firm may not obtain a rating from agency 1 because this scenario improves the average quality of the firms rated by agency 1. Thus, agency 1 lies for sure, as under monopoly, and is able to leave a positive surplus for a bad firm. In addition, agency 1 strictly prefers to rate a bad firm at any $\phi_1^b > \delta V(1)$ because rating a bad firm gives $w(p^{G_1}) > \delta V(1)$, which is at least as good as the discounted future value. So, a bad firm must obtain a rating from agency 1 with probability 1. Thus, the value of agency 1's good rating is $w(p^{G_1}(q)) = w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-q_1)}\right)$, which is the same as when agency 1 is the monopolist. Because the reputation profile does not change when the firm is good,

$$\begin{aligned} V_1(q) &\leq \frac{w(p^{G_1}(q)) - \lambda\delta V_2(q)}{1-\lambda\delta} \\ &< V(q_1). \end{aligned}$$

■

Now I can prove lemma 14.

Proof for Lemma 14. If $q_1 \geq \rho_1^M$, then by claim 16 and inequality (14),

$$V_1(q) \leq \frac{w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-q_1)}\right) - \lambda\delta V_2(q)}{1-\lambda\delta}.$$

Because $V_2(q) > 0$, $V_1(q) < V(q_1)$. ■

For $q_1 < \rho_1^M$, when the project is bad, there can be only zero marginal surplus from agency 1's good rating because if agency 1 can give a positive surplus to a bad firm, by the zero surplus lemma (10) , it will lie for sure, and $\chi_1^{B_1} = 1$. But then a bad firm will approach agency 1 with probability 1; thus

$$\begin{aligned} w(p^{G_1}) &= w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-q_1)}\right) \\ &< \delta V(1) \\ &= \delta V_1(\chi^{B_1}), \end{aligned}$$

a contradiction. Hence, in equilibrium, agency 1's good rating has zero marginal surplus when the firm is bad, i.e.,

$$\begin{aligned} w(p^{G_1}(q)) &= w\left(\frac{\lambda}{\lambda+(1-\lambda)\beta_1(q)(1-a_1^*(q))}\right) \\ &= \delta V_1\left(\frac{q_1}{a_1^*(q)}, q_2\right). \end{aligned}$$

In addition, agency 2's good rating must also have zero marginal surplus when the firm is bad; otherwise agency 2 will rate bad firms and only bad firms in equilibrium and $\lambda_2 = 0$, a contradiction to hypothesis that there is positive surplus. Thus, the discount that agency 1 gives a good firm is $\delta V_2(q)$. Because agency 1 is indifferent between lying and not lying, its equilibrium payoff is

$$V_1(q) = \frac{(\lambda + (1-\lambda)\beta_1) * w\left(p^{G_1}(q)\right) - \lambda\delta V_2(q)}{1 - \lambda\delta - (1-\lambda)(1-\beta_1)\delta}.$$

Because $\frac{\delta}{1-\lambda\delta} < 1$, to show that $V_1(q) < V(q_1)$, it suffices to show that the value of agency 1's good rating is lower under duopoly.

Suppose there exists $q_1 > q_2$, where $(q_1, q_2) \in (0,1)$, such that

$$V_1(q) \geq V(q_1).$$

We can construct a sequence $\left\{\left(q_1^k, q_2^k\right)\right\}_k$ such that this holds for all k and $q^{k+1} > q^k$. This sequence is finite if there exists K such that $V_1(q) < V(q_1)$ for all $q > q^K$. Define $\overline{q} := q^K$ if it is finite and $\overline{q} := \lim_{k\to\infty} q^k$ if the sequence is infinite.

By lemma 8, $a^*(\overline{q}) < 1$. Thus, there exists $\hat{q} \in \left\{q^k\right\}$ such that $\hat{q}_1 \in (\overline{q}_1 * a^*(\overline{q}_1), \overline{q}_1]$ and $V_1(\hat{q}) \geq V(\hat{q}_1)$. For all $a_1 \geq a^*(\overline{q}_1)$, where $a^*(\overline{q}_1)$ is the equilibrium accuracy when the monopolist's reputation is$\overline{q}_1$

$$\begin{aligned}
w\left(\frac{\lambda}{\lambda + (1-\lambda)(1-a_1)}\right) &\geq w\left(\frac{\lambda}{\lambda + (1-\lambda)(1-a^*(\overline{q}_1))}\right) \\
&= \delta V\left(\frac{\overline{q}_1}{a^*(\overline{q}_1)}\right) \\
&> \delta V\left(\frac{\hat{q}_1}{a^*(\overline{q}_1)}\right) \\
&> \delta V\left(\frac{\hat{q}_1}{a^*(\overline{q}_1)}, \hat{q}_2\right)
\end{aligned}$$

because $\frac{\hat{q}_1}{a^*(\overline{q}_1)} > \overline{q}_1$ and by definition of $\overline{q}$. So, $a^*(\hat{q}_1, \hat{q}_2) < a^*(\overline{q}_1)$.

If $\beta_1(\hat{q}) = 1$, then $w\left(p^{G_1}(\hat{q})\right) = w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-a^*(\hat{q}))}\right) < w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-a^*(\overline{q}_1))}\right)$. It follows that $V_1(\hat{q}) < V(\hat{q}_1)$. If $\beta_1(\hat{q}) < 1$, by the maximum coverage condition, then agency 1 must weakly prefers the bad firm to obtain no rating or to approach agency 2. If no rating is issued, then the reputation profile remains unchanged. Thus, if agency 1 prefers no rating to be issued to rating a bad firm, then

$$\begin{aligned}
V_1(\hat{q}) &\leq \frac{(\lambda + (1-\lambda)\beta_1) w\left(p^{G_1}(\hat{q})\right)}{1 - \lambda\delta - (1-\lambda)(1-\beta_1)\delta} \\
&\leq \frac{(\lambda + (1-\lambda)\beta_1)\delta V_1(\hat{q})}{1 - \lambda\delta - (1-\lambda)(1-\beta_1)\delta} \\
&< V_1(\hat{q}),
\end{aligned}$$

a contradiction. Thus agency 1 must weakly prefer a bad firm to obtain a rating from agency 2. Hence,

$$\begin{aligned} V_1(\hat{q}) &\leq \frac{(\lambda + (1-\lambda)\beta_1) w\left(p^{G_1}(\hat{q})\right)}{1 - \lambda\delta - (1-\lambda)(1-\beta_1)\delta} \\ &\leq \left(1 - \frac{1 - \delta(1+\lambda)}{1 - \delta + (1-\lambda)\delta}\right)\left[(1 - a_2^*(\hat{q})) V(\hat{q}_1) + a_2^*(\hat{q}) V\left(\hat{q}_1, \frac{\hat{q}_2}{a_2^*(\hat{q})}\right)\right]. \end{aligned}$$

If $\frac{\hat{q}_2}{a_2^*(\hat{q})} > \hat{q}_1$, then by claim 34, $V\left(\hat{q}_1, \frac{\hat{q}_2}{a_2^*(\hat{q})}\right) < V(\hat{q}_1)$ and we are finished. Therefore, $V_1(\hat{q}) \geq V(\hat{q}_1)$ only if $V_1\left(\hat{q}_1, \frac{\hat{q}_2}{a_2^*(\hat{q})}\right) > V(\hat{q}_1)$ and $\frac{\hat{q}_2}{a_2^*(\hat{q})} \leq \hat{q}_1$. Then, we can construct a sequence $\left\{\hat{q}^k\right\}_k$ where $\hat{q}_1^k = \hat{q}_1$ for all k and $\hat{q}_2^0 = \hat{q}_2$ and $\hat{q}_2^{k+1} = \frac{\hat{q}_2^k}{a_2^*(\hat{q}^k)} \leq \hat{q}_1$ such that $V_1\left(\hat{q}^{k+1}\right) > V_1\left(\hat{q}^k\right) \geq V(\hat{q}_1)$ for all $k \geq 0$. Then $\lim_{k\to\infty} a_2^*\left(\hat{q}^k\right) = \lim_{k\to\infty} \frac{\hat{q}_2^k}{\hat{q}_2^{k+1}} = 1$ and $\lim_{k\to\infty} V_1\left(\hat{q}^k\right) > V(\hat{q}_1)$. Thus,

$$\lim_{k\to\infty} V_1\left(\hat{q}^k\right) \leq \left(1 - \frac{1 - \delta(1+\lambda)}{1 - \delta + (1-\lambda)\delta}\right) \lim_{k\to\infty} V_1\left(\hat{q}^{k+1}\right).$$

But $\frac{1-\delta(1+\lambda)}{1-\delta+(1-\lambda)\delta} > 0$, a contradiction. Hence, there exists no $q_1 \geq q_2$ such that $V_1(q_1, q_2) \geq V(q_1)$.

A.5 Proofs for Equilibrium Construction

A.5.1 Behaviors On the Equilibrium Path

Proof for Claim 19. The LHS of (23) is continuously increasing and the right hand side is continuously decreasing in a^*. In addition,

$$w\left(\frac{\lambda}{\lambda + (1-\lambda)\left(1 - \frac{\rho_{k+1}^D(q_2)}{\rho_k^D(q_2)}\right)}\right) = \delta\tilde{V}\left(\rho_k^D, q_2\right) < \delta\tilde{V}\left(\frac{q_1}{\rho_{k+1}^D(q_2)}\rho_k^D\right),$$

and strict inequality holds in the opposite direction when $a^* = \tilde{a}^*\left(\rho_k^D(q_2), q_2\right) = \frac{\rho_k^D(q_2)}{\rho_{k-1}^D(q_2)}$. Thus, a unique solution exists and belongs to

$$\left[\tilde{a}^*\left(\rho_{k+1}^D(q_2), q_2\right), \tilde{a}^*\left(\rho_k^D(q_2), q_2\right)\right].$$

It follows that $\frac{q_1}{\tilde{a}^*(q_1,q_2)} > \rho_k^D(q_2)$. By hypothesis, the RHS decreases with q_2 and increases in q_1. Thus, $\tilde{a}^*(q_1, q_2)$ decreases in q_2 and increases in q_1. In addition, $\frac{q_1}{\tilde{a}^*(q_1,q_2)} = \tilde{\chi}^B(q_1, q_2)$ increases with q_1 and decreases with q_2. ■

Claim 35 *$\phi(r)$ is well-defined and continuous for all $r \in I$.*

Proof. The left hand side is strictly increasing in ϕ because the reputation earned by giving a bad rating, $\frac{\phi}{\tilde{a}^*(\phi,r)} = \chi_1^{B_1}(\phi, r)$, is strictly increasing in the agency's own reputation. The right hand side is strictly decreasing in ϕ because

$$
\begin{aligned}
& \frac{\partial}{\partial\phi}\left\{\begin{array}{c} \frac{1-(1-\lambda)\delta+(1-\lambda)\delta\beta_1}{\lambda+(1-\lambda)\delta\beta_1}\frac{(1-\lambda)\delta}{1-\lambda\delta-(1-\lambda)\delta+(1-\lambda)\delta\beta_1} \\ \times\left(1-a(\phi,r)\right)\left[V(r)+\frac{a(\phi,r)\frac{r}{\phi}}{1-a(\phi,r)\frac{r}{\phi}}V\left(r,\frac{\phi}{a(\phi,r)}\right)\right] \end{array}\right\} \\
= & -\frac{1-(1-\lambda)\delta+(1-\lambda)\delta\beta_1}{\lambda+(1-\lambda)\delta\beta_1}\frac{(1-\lambda)\delta}{1-\lambda\delta-(1-\lambda)\delta+(1-\lambda)\delta\beta_1} \\
& \times\left(\frac{((1-\lambda)\delta)}{(\lambda+(1-\lambda)\delta\beta_1)(1-\lambda\delta-(1-\lambda)\delta+(1-\lambda)\delta\beta_1)}\right) \\
& \times\left(\begin{array}{c} (1-\lambda)(1-\delta)\frac{(1-\lambda\delta-(1-\lambda)\delta+(1-\lambda)\delta\beta_1)}{1-(1-\lambda)\delta+(1-\lambda)\delta\beta_1} \\ +\frac{(1-(1-\lambda)\delta+(1-\lambda)\delta\beta_1)(\lambda+(1-\lambda)\delta\beta_1)}{1-(1-\lambda)\delta+(1-\lambda)\delta\beta_1} \end{array}\right) \\
& \times\left(\frac{-\left(1-\frac{r}{\phi}\right)\frac{\partial a}{\partial\phi}+(1-a)\frac{r}{\phi}\frac{a}{\phi}}{\left(1-a(\phi(r),r)\frac{r}{\phi(r)}\right)^2}\right)(1-a) \\
& \times\left[V(r)+\frac{a(\phi,r)\frac{r}{\phi}}{1-a(\phi,r)\frac{r}{\phi}}V\left(r,\frac{\phi}{a(\phi,r)}\right)\right] \\
& -\left\{\begin{array}{c} \frac{1-(1-\lambda)\delta+(1-\lambda)\delta\beta_1}{\lambda+(1-\lambda)\delta\beta_1}\frac{(1-\lambda)\delta}{1-\lambda\delta-(1-\lambda)\delta+(1-\lambda)\delta\beta_1} \\ \times\left[\frac{\partial a}{\partial\phi}\left(V(r)+\frac{a(\phi,r)\frac{r}{\phi}}{1-a(\phi,r)\frac{r}{\phi}}V\left(r,\frac{\phi}{a(\phi,r)}\right)\right)+..\right] \end{array}\right\} \\
< & \frac{1-(1-\lambda)\delta+(1-\lambda)\delta\beta_1}{\lambda+(1-\lambda)\delta\beta_1}\frac{(1-\lambda)\delta}{1-\lambda\delta-(1-\lambda)\delta+(1-\lambda)\delta\beta_1} \\
& \times\frac{\partial a}{\partial\phi}\left(V(r)+\frac{a(\phi,r)\frac{r}{\phi}}{1-a(\phi,r)\frac{r}{\phi}}V\left(r,\frac{\phi}{a(\phi,r)}\right)\right) \\
& \times\left[\frac{1-a}{1-a\frac{r}{\phi}}\frac{1-\frac{r}{\phi}}{1-a\frac{r}{\phi}}\left(\begin{array}{c} (1-\lambda)(1-\delta)\frac{(1-\lambda\delta-(1-\lambda)\delta+(1-\lambda)\delta\beta_1)}{1-(1-\lambda)\delta+(1-\lambda)\delta\beta_1} \\ +\frac{(1-(1-\lambda)\delta+(1-\lambda)\delta\beta_1)(\lambda+(1-\lambda)\delta\beta_1)}{1-(1-\lambda)\delta+(1-\lambda)\delta\beta_1} \end{array}\right)-1\right] \\
< & \frac{1-(1-\lambda)\delta+(1-\lambda)\delta\beta_1}{\lambda+(1-\lambda)\delta\beta_1}\frac{(1-\lambda)\delta}{1-\lambda\delta-(1-\lambda)\delta+(1-\lambda)\delta\beta_1} \\
& \times\frac{\partial a}{\partial\phi}\left(V(r)+\frac{a(\phi,r)\frac{r}{\phi}}{1-a(\phi,r)\frac{r}{\phi}}V\left(r,\frac{\phi}{a(\phi,r)}\right)\right) \\
& \times\left[(1-\lambda)(1-\delta)+\lambda+(1-\lambda)\delta\beta_1-1\right] \\
< & \ 0.
\end{aligned}
$$

When $r \in I$, LHS<RHS for $\phi = \sup\left\{q_1 : \tilde{V}(q_1, q_2) \leq \tilde{V}(q_2, q_1)\right\}$, and LHS > RHS for $\phi = \rho_1^M$. So there exists a unique $\phi(r)$ such that equality holds. In addition, by the implicit function theorem, $\phi(r)$ is continuous in r. ∎

Claim 36 *Agency i is indifferent between lying and not lying at (r, r), for any $r \in (0, 1)$.*

Proof. By definition and continuity, $p^G(r,r) = \lim_{q_1 \to r^+} p^G(q_1, r)$ and $\chi^B(r,r) = \lim_{q_1 \to r^+} \chi^B(q_1, r)$. Thus

$$\begin{aligned} & w\left(\frac{\lambda}{\lambda + (1-\lambda)\beta(r,r)(1-a^*(r,r))}\right) \\ = & \lim_{q_1 \to r^+} w\left(\frac{\lambda}{\lambda + (1-\lambda)\beta(q_1,r)(1-a^*(q_1,r))}\right) \\ = & \lim_{q_1 \to r^+} \delta V\left(\frac{q_1}{a^*(q_1,r)}, r\right) \\ = & \delta V\left(\frac{r}{a^*(r,r)}, r\right). \end{aligned}$$

The last equality is established by continuity in lemma 21. Hence, at $x^*(r,r)$ so defined, agency 1 is indifferent between lying and not lying. ■

A.5.2 Properties of the Value Function

Claim 37 *$\frac{\partial V}{\partial q_1}(q_1,q_2) > 0$, but $\frac{\partial V}{\partial q_1}(q_2,q_1) < 0$ for $(q_1,q_2) \in A$.*

Proof. Consider (q,r) where $r \notin I$ or $r \in I$ but $q \geq \phi(r)$. Then $a^*(q,r) = \tilde{a}^*(q,r)$. Because $\tilde{a}^*(q,r)$ is the intersection of $\delta V\left(\frac{q}{a}, r\right)$ and $w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-a)}\right)$, it increases with q, and $\frac{q}{a^*(q,r)}$ increases with q. Because $V\left(r, \frac{r}{a^*(q,r)}\right) < V(r)$, $V(r,q)$ is decreasing in q if it is decreasing in the second argument at $\frac{q}{a^*(q,r)}$. For $q \in \left[\rho_1^M, 1\right]$, $a^*(q,r) = q$, $V(r,q) = \frac{(1-\lambda)\delta}{1-\lambda\delta}(1-q)V(r)$ is decreasing in q. Then the proof for this case is concluded by induction. Because $w\left(p^G(q,r)\right) = w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-a^*(q,r))}\right)$ increases with q and $V(r,q)$ decreases with q, $V(q,r)$ increases with q.

Consider $r \in I$ and $q \in (r, \phi(r)]$. Then

$$\begin{aligned} V(r,q) = & \left[\lambda\delta + (1-\lambda)\delta(1-\beta^*(q,r))\right]V(r,q) \\ & + (1-\lambda)\delta\left[(1-\tilde{a}^*(\phi(r),r))V(r) + \tilde{a}^*(\phi,r)\frac{q}{\phi(r)}\beta^*(q,r)V\left(r, \frac{\phi(r)}{a^*(\phi(r),r)}\right)\right] \end{aligned}$$

Taking derivative w.r.t. q on both sides, and then re-arranging, we get

$$\begin{aligned} & \frac{\partial V(r,q)}{\partial q}\left(1 - \lambda\delta - (1-\lambda)\delta(1-\beta^*(q,r))\right) \\ = & -(1-\lambda)\delta\frac{\partial\beta^*(q,r)}{\partial q}V(r,q) \\ & + (1-\lambda)\delta\tilde{a}^*(\phi,r)\frac{q}{\phi(r)}\frac{\partial\beta^*(q,r)}{\partial q}V\left(r, \frac{\phi(r)}{\tilde{a}^*(\phi(r),r)}\right) \\ & + (1-\lambda)\delta\tilde{a}^*(\phi,r)\frac{1}{\phi(r)}\beta^*(q,r)V\left(r, \frac{\phi(r)}{\tilde{a}^*(\phi(r),r)}\right) \end{aligned}$$

Because

$$\frac{\partial\beta^*(q,r)}{\partial q}=\beta^*(q,r)\frac{\frac{\tilde a^*(\phi,r)}{\phi}}{1-\tilde a^*(\phi(r),r)\frac{q}{\phi(r)}}$$

we have

$$\begin{aligned}
&\frac{\partial V(r,q)}{\partial q}\frac{(1-\lambda\delta-(1-\lambda)\delta(1-\beta^*(q,r)))}{(1-\lambda)\delta\frac{\partial\beta^*(q,r)}{\partial q}}\\
=\;&\left(1-\tilde a^*(\phi(r),r)\frac{q}{\phi(r)}\right)V\left(r,\frac{\phi}{a^*(\phi,r)}\right)\\
&+\tilde a^*(\phi,r)\frac{q}{\phi(r)}V\left(r,\frac{\phi}{a^*(\phi,r)}\right)-V(r,q)\\
=\;&V\left(r,\frac{\phi}{a^*(\phi,r)}\right)-V(r,q)
\end{aligned}$$

Plugging $q=\phi(r)$ and q respectively and substract one from the other, we get

$$\begin{aligned}
V(r,\phi(r))-V(r,q)\;=\;&\lambda\delta(V(r,\phi(r))-V(r,q))\\
&+(1-\lambda)\delta\tilde a^*(\phi,r)\left(1-\frac{q}{\phi}\beta^*(q,r)\right)V\left(r,\frac{\phi}{a^*(\phi,r)}\right)\\
&-(1-\lambda)\delta(1-\beta^*(q,r))V(r,q)\\
=\;&\lambda\delta(V(r,\phi(r))-V(r,q))\\
&+(1-\lambda)\delta(1-\beta^*(q,r))\left(V\left(r,\frac{\phi}{a^*(\phi,r)}\right)-V(r,q)\right)
\end{aligned}$$

because $\tilde a^*(\phi,r)\left(1-\frac{q}{\phi}\beta^*(q,r)\right)=1-\beta^*(q,r)$ by rearranging the definition of $\beta^*(q,r)$. So

$$V(r,\phi(r))-V(r,q)=\frac{(1-\lambda)\delta}{1-\lambda\delta}(1-\beta^*(q,r))\left(V\left(r,\frac{\phi}{a^*(\phi,r)}\right)-V(r,q)\right)$$

So

$$\begin{aligned}
V(r,\phi(r))-V(r,q)\;=\;&\frac{\frac{(1-\lambda)\delta}{1-\lambda\delta}(1-\beta^*(q,r))}{1-\frac{(1-\lambda)\delta}{1-\lambda\delta}(1-\beta^*(q,r))}\left(V\left(r,\frac{\phi}{a^*(\phi,r)}\right)-V(r,\phi(r))\right)\\
<\;&0
\end{aligned}$$

because So $\frac{\partial V(r,q)}{\partial q}<0$ for $q\in(r,\phi(r)]$.

For $q\in(r,\phi(r)]$,

$$V(q,r)=\frac{(\lambda+(1-\lambda)\beta^*(q,r))w\left(p^G(\phi(r),r)\right)-\lambda\delta V(r,q)}{1-\delta+(1-\lambda)\delta(1-\beta^*(q,r))}.$$

It's straightforwad to see that $V(q,r)$ increases with $\beta^*(q,r)$ because $1-\delta-\lambda\delta > 0$. Because $\beta^*(q,r)$ increases with q and $V(r,q)$ decreases with q, $V(q,r)$ increases with q.

The value of business for the leading agency is

$$V(q_1,q_2) = \frac{(\lambda + (1-\lambda)\beta_1^*) w\left(\frac{1}{1+\frac{1-\lambda}{\lambda}(1-a^*(\phi(q_2),q_2))}\right) - \lambda\delta V(q_2,q_1)}{1-\delta+(1-\lambda)\beta_1^*\delta}.$$

Given the competitor's reputation, the value of the leading agency's good rating does not change with its own reputation, for $q \in (q_2, \phi(q_2))$. So, the fee it charges a bad firm does not change with its own reputation, but the probability of it being approached by a bad firm increases with its own reputation. In addition, the fee it charges a good firm increases with its own reputation, because when its reputation is better, the competitor derives a lower value and thus is less of a threat in competing for a good firm. We can immediately see that the leading agency's value increases with its own reputation. ■

Claim 38 *For $r \in I$, $\lim_{q_1 \to r^+} V(q_1,r) = \lim_{q_2 \to r^-} V(q_2,r) = V(r,r)$.*

Proof. Write $\beta_1(r^+,r) = \frac{1-a^*(\phi(r),r)}{1-a^*(\phi(r),r)\frac{r}{\phi}}$. By the construction of $\phi(r)$,

$$\begin{aligned}
& \lim_{q_1 \to r^+} (V(q_1,r) - V(r,q_1)) \\
= & \frac{(\lambda + (1-\lambda)\beta_1(r^+,r))\delta V\left(\frac{\phi}{\tilde{a}^*(\phi(r),r)}, r\right) - \lambda\delta \lim_{q_1 \to r^+} V(r,q_1)}{1-\delta+(1-\lambda)\beta_1(r^+,r)\delta} \\
& - \lim_{q_1 \to r^+} V(r,q_1) \\
= & \frac{\left\{\begin{array}{c} (1-(1-\lambda)(1-\beta_1(r^+,r)))\delta V\left(\frac{\phi}{\tilde{a}^*(\phi(r),r)}, r\right) \\ -(1-(1-\lambda)(1-\beta_1(r^+,r))\delta)\lim_{q_1 \to r^+} V(r,q_1) \end{array}\right\}}{1-\delta+(1-\lambda)\beta_1(r^+,r)\delta} \\
= & 0.
\end{aligned}$$

Thus, $V(r,r) = \lim_{q_1 \to r^+} V(q_1,r)$. Because both $\phi(.)$ and $\tilde{a}^*(.,.)$ are continuous,

$$\begin{aligned}
\lim_{q_2 \to r^-} \beta_1(r,q_2) &= \frac{1-a(\phi(q_2),q_2)}{1-a(\phi(q_2),q_2)\frac{r}{\phi(q_2)}} \\
&= \frac{1-a(\phi(r),r)}{1-a(\phi(r),r)\frac{r}{\phi(r)}} = \lim_{q_1 \to r^+} \beta(q_1,r).
\end{aligned}$$

Thus,

$$
\begin{aligned}
& \lim_{q_2 \to r^-} V(r, q_2) \\
= & \lim_{q_2 \to r^-} \left\{ \begin{array}{c} \frac{(1-\lambda)\delta}{1-\delta+(1-\lambda)\beta_1(r,q_2)\delta} \\ \times \left(\begin{array}{c} (1-a(\phi(q_2), q_2)) V(q_2) \\ +\frac{(1-a(\phi(q_2),q_2))a(\phi(q_2),q_2)\frac{r}{\phi(q_2)}}{1-a(\phi(q_2),q_2)\frac{r}{\phi(q_2)}} V\left(q_2, \frac{\phi(q_2)}{a(\phi(q_2),q_2)}\right) \end{array} \right) \end{array} \right\} \\
= & \frac{(1-\lambda)\delta}{1-\delta+(1-\lambda)\left(\lim_{q_1 \to r^+} \beta(q_1, r)\right)\delta} \\
& \times \left[\begin{array}{c} (1-a(\phi(r), r)) V(r) \\ +\frac{(1-a(\phi(r),r))a(\phi(r),r)\frac{r}{\phi(r)}}{1-a(\phi(r),r)\frac{r}{\phi(r)}} \end{array} \right] V\left(r, \frac{\phi(r)}{a(\phi(r), r)}\right) \\
= & \lim_{q_1 \to r^+} V(q_1, r).
\end{aligned}
$$

■

A.5.3 Behaviors Off the Equilibrium Path

Lemma 40 in Appendix A.5.5 shows that we can construct $(\lambda(q_2, q_1), x^*(q_2, q_1))$, the trailing agency's probability of lying, and investors' belief about projects rated by the trailing agency, such that agency 2 can leave only zero surplus for a bad firm and both types of agency 2 prefer a bad firm to approach it to the firm obtaining no rating.

I will finish the description of the equilibrium by establishing the players' behaviors when the proposed fee profile is not on the equilibrium path.

When the firm with a type v project approaches agency i after i proposes $\phi_i \neq \phi_i^v$, agency i gives a good rating if and only if $\phi_i \geq c_i^v$.

Given a fee profile (ϕ_1, ϕ_2) and $y(.)$ defined previously, by approaching agency i, a good firm expects to receive a surplus of

$$
U_i^g(\phi; q) = \left\{ \begin{array}{ccc} w\left(p^{G_i}\right) - \phi_i & \text{if} & \phi_i \geq c_i^g \\ \gamma(\phi_i; q) * \left(w\left(p^{G_i}\right) - \phi\right) & \text{if} & \phi_i < c_i^g \end{array} \right. ,
$$

because when the rating fee is below agency i's cost, only an honest agency i will give a good rating. Let a good firm's belief about agency i be given by

$$
\gamma^g(\phi_i; q) = \left\{ \begin{array}{cc} q_i & \text{if } \phi_i \geq c^g(q_i, q_j) \\ \min\left\{ q_i, \frac{\delta V(q_i,q_j)\frac{q_i+1}{2}}{w(p^G(q_i,q_j))-\phi_i} \right\} & \text{otherwise} \end{array} \right. .
$$

It is continuous in ϕ_i. Let $\alpha(.)$ be such that $\alpha(\phi_1, \phi_2; q) = 1$ if $U_1^g(\phi; q) \geq \max\{U_2^g(\phi; q), 0\}$ and $\alpha(\phi_2, \phi_1; q) = 1$ if $U_2^g(\phi; q) > U_1^g(\phi; q)$ and $U_2^g(\phi; q) \geq 0$, and $\alpha(\phi_1, \phi_2; q) = \alpha(\phi_2, \phi_1; q) = 0$ otherwise.

Let a bad firm's belief about firm i to be $\gamma^b(\phi_i; q) = q_i$ for all $\phi_i \geq 0$. If agency 1 charges $\phi_1 = w\left(p^G(q_1, q_2)\right)$, then a bad firm approaches it with probability $\beta^*(q_1, q_2)$, and obtains no rating with probability $1 - \beta^*(q_1, q_2)$. If agency 1 charges $\phi_1 \in [\delta V\left(\chi^B(q_1,q_2), w\left(p^G(q_1,q_2)\right)\right)$ where $\phi_1 \neq \phi^b(q_1,q_2)$, then the bad firm obtains a rating from it with probability 1. Otherwise, the bad firm obtains a rating from agency 1 with probability 0. In addition, if $\phi_2 \in [\delta V\left(\chi^B(q_2,q_1), w\left(p^G(q_2,q_1)\right)\right)$ and $\phi_2 \neq \phi^b(q_2,q_1)$, then the bad firm obtains a rating from agency 2 with probability 1, but if $\phi_2 = w\left(p^G(q_2,q_1)\right)$, then it does so with probability $\varepsilon(q_1,q_2)$ such that

$$\varepsilon V^t(1) + (1-\varepsilon) V^t(q_1,q_2) < V^t\left(\frac{q_1}{a^*(q_1,q_2)}, q_2\right), \tag{28}$$

where V^t denotes the equilibrium payoff for a type t agency 1. Such an ε exists because the value for both types of the leading agency value is strictly increasing in its own reputation (lemma 37 in Section 3.6.3 and lemma 41 in Appendix A.5.5).

A.5.4 Verification of Equilibrium Conditions

By construction, $x^*$and y^* are optimal for an agency given the equilibrium fee. $x(.)$ and $y(.)$ are constructed to maximize an agency's intertemporal payoffs.

Because $V(q_1,q_2) > V(q_2,q_1)$ and agency 2 cannot leave positive a surplus for a bad firm, α^* and β^* are a good and bad firm's best response at the equilibrium fee profile. The behaviors $\alpha(.)$ and $\beta(.)$ for off-the-equilibrium-path fee profiles are constructed to be optimal.

By construction of $\lambda(q_2,q_1)$, agency 2 weakly prefers a bad firm to obtain a rating from himself to it obtaining no rating. Thus the rating fee agency 2 charges as defined by (18) is nonnegative. Because the surplus a good firm expects from agency 2 is maximized when agency 2 charges its equilibrium fee, $\phi^g(q_2,q_1)$ is optimal and weakly undominated for agency 2. Because a bad firm does not approach agency 2 when agency 1 charges its equilibrium fee, agency 2 is indifferent between all fees it charges. By the construction of $w\left(p^G(q_2,q_1)\right)$ and $\beta(\phi;q_2,q_1)$, charging (19) to a bad firm is not weakly dominated for agency 2 because agency 2 weakly prefers rating a bad firm to the outcome when no rating is given.

When the firm is good, on the equilibrium path, agency 1 receives a positive rating fee by giving a good rating because

$$\begin{aligned}\delta V\left(\chi^B(q_1,q_2)\right) &= \delta V\left(\frac{q_1}{a^*(q_1,q_2)}, q_2\right) \\ &> \delta V(q_1,q_2) \\ &> \delta V(q_2,q_1),\end{aligned}$$

and the future reputation profile is the same no matter which agency rates the good firm. Thus, $\phi^g(q_1,q_2)$ defined by (18) is optimal for both types of agency 1. When the firm is bad, by charging the equilibrium fee, agency 1 has the option of giving a bad rating and earning future value from reputation profile $\left(\frac{q_1}{a^*(q_1,q_2)}, q_2\right)$. Given that agency 2 charges the equilibrium fee, by leaving a smaller surplus for the bad firm, the firm will obtain no rating with probability $1-\varepsilon(q_1,q_2)$, and a rating from agency 2 with probability $\varepsilon(q_1,q_2)$. By (28) and the construction of $\beta(.)$, the fee $\phi^b(q_1,q_2)$ defined by (19) is optimal for both types of agency 1.

By lemma 22, the leading agency always has a higher value than the trailing agency. The following claim implies that when the firm has a bad project, for all $q \in A$, agency 1 prefers the firm to obtain a rating from the trailing agency. Because $\beta^*(q) = 1$ for all $q \notin A$, maximum coverage condition is also satisfied.

Claim 39 *For all $q_2 \in I$ and $q_1 \in (q_2, \phi(q_2))$,*

$$w\left(p^G(q_1,q_2)\right) < (1-a^*(q_2,q_1))\,V(q_1) + a^*(q_2,q_1)\,V\left(q_1, \frac{q_2}{a^*(q_2,q_1)}\right).$$

Proof. By definition,

$$\begin{aligned}
& w\left(p^G(q,r)\right) \\
= \;& \delta V\left(\frac{\phi}{a(\phi,r)}, r\right) \\
= \;& \frac{(1-(1-\lambda)(1-\beta_1(r^+,r))\delta)}{(1-(1-\lambda)(1-\beta_1(r^+,r)))}\frac{(1-\lambda)}{1-\lambda\delta-(1-\lambda)(1-\beta_1(r^+,r))\delta} \\
& \times\left[(1-a(\phi,r))\,\delta V(r) + \frac{(1-a(\phi,r))\,a(\phi,r)\,\frac{r}{\phi}}{1-a(\phi,r)\,\frac{r}{\phi}}\delta V\left(r, \frac{\phi}{a(\phi,r)}\right)\right] \\
< \;& \left(\frac{(1-(1-\lambda)(1-\beta_1(r^+,r))\delta)}{(1-(1-\lambda)(1-\beta_1(r^+,r)))}\frac{(1-\lambda)\beta_1(r^+,r)}{1-\lambda\delta-(1-\lambda)(1-\beta_1(r^+,r))\delta}\right) \\
& \times(1-r)\,\delta V(r) \\
< \;& (1-r)\,\delta V(r).
\end{aligned}$$

because

$$\begin{aligned}
& \frac{(1-(1-\lambda)(1-\beta_1(r^+,r))\delta)}{(1-(1-\lambda)(1-\beta_1(r^+,r)))}\frac{(1-\lambda)\beta_1(r^+,r)}{1-\lambda\delta-(1-\lambda)(1-\beta_1(r^+,r))\delta} \\
= \;& \left(1+\frac{(1-\lambda)(1-\beta_1(r^+,r))(1-\delta)}{(1-(1-\lambda)(1-\beta_1(r^+,r)))}\right)\left(1-\frac{(1-\delta)(1-(1-\lambda)\beta_1)}{1-\delta+(1-\lambda)\delta\beta_1(r^+,r)}\right) \\
= \;& \left(1+\frac{(1-\lambda)(1-\beta_1(r^+,r))(1-\delta)}{(1-(1-\lambda)(1-\beta_1(r^+,r)))}\right)\left(1-\frac{(1-\delta)(\lambda+(1-\lambda)(1-\beta_1))}{1-\delta+(1-\lambda)\delta\beta_1(r^+,r)}\right) \\
< \;& 1
\end{aligned}$$

Because $a_2^* \geq r$ and $V(q_1,q_2') < V(q_1)$ for all q_2', $w\left(p^G(q,r)\right) < (1-a_2^*)\,\delta V(q) + a_2^*\delta V\left(q, \frac{r}{a_2^*}\right)$. ■

In equilibrium, an agency gives nonnegative surplus to a firm, and a firm is indifferent between both agencies. In addition, $\lambda(q_i,q_j) \in (0,1)$ for all $(q_i,q_j) \in [0,1]^2$. Consistency follows immediately.

Thus, the strategy profiles and beliefs constructed constitute an equilibrium in which the leading agency has higher value.

A.5.5 More Detailed Proofs for Behaviors Off The Equilibrium Path Conditions

Lemma 40 *There exists* $\lambda(q_2, q_1)$ *and* $a^*(q_2, q_1)$ *on* C *such that*

$$w\left(\frac{\lambda_2(q_2,q_1)}{\lambda(q_2,q_1)+(1-\lambda(q_2,q_1))(1-a^*(q_2,q_1))}\right)=\delta V\left(\frac{q_2}{a^*(q_2,q_1)},q_1\right),$$

$$V\left(\frac{q_2}{a^*(q_2,q_1)},q_1\right)\geq V(q_2,q_1),$$

and

$$V^h\left(\frac{q_2}{a^*(q_2,q_1)},q_1\right)\geq V^h(q_2,q_1).$$

Proof. For $q_1 > q_2 > \rho_1^M$, define $\lambda(q_2, q_1) = \lambda_2$ such that $w\left(\frac{\lambda_2}{\lambda_2+(1-\lambda_2)(1-q_2)}\right) = \delta V(1)$. So,

$$\lambda_2=\frac{1}{1+\frac{\left(w^{-1}\left(\frac{1}{\delta V(1)}\right)\right)^{-1}-1}{1-q_2}}.$$

Also define $x^*(q_2, q_1) = 1$.

For $q_1 \notin I$, define $\psi(q_1)$ to be $\hat{q}_2$ such that

$$w\left(\frac{\lambda}{\lambda+(1-\lambda)\left(1-\frac{\hat{q}_2}{q_1}\right)}\right)=\lim_{q_1'\to q_1^+}\delta V(q_1',q_1).$$

Then, for $q_2 \in (\psi(q_1), q_1)$, define $\tilde{\lambda}(q_2, q_1) = \lambda$, and $\tilde{a}^*(q_2, q_1)$ to be the solution to

$$w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-a)}\right)=\delta V\left(\frac{q_2}{a},q_1\right).$$

Because $q_2 > \psi(q_1)$, and $V\left(\frac{q_2}{a}, q_1\right)$ is decreasing in a for all $a < \frac{q_2}{q_1}$, a unique solution in $\left(q_2, \frac{q_2}{q_1}\right)$ exists. Define $\tilde{a}^*(q_2, q_1) = \frac{q_2}{q_1}$, and $\tilde{\lambda}(q_2, q_1) = \lambda_2$ to be such that

$$w\left(\frac{\lambda_2}{\lambda_2+(1-\lambda_2)\left(1-\frac{\psi(q_1)}{q_1}\right)}\right)=\delta V(q_1,q_1).$$

For $q_2 < \psi(q_1)$, define $\tilde{\lambda}(q_2, q_1) = \lambda_2$ to be the solution to

$$w\left(\frac{\lambda_2}{\lambda_2+(1-\lambda_2)\left(1-\frac{\psi(q_1)}{q_1}\right)}\right)=\lim_{q_2'\to q_1^-}\delta V(q_2',q_1).$$

Define $\tilde{a}^*(q_2, q_1)$ to be the solution a^* to

$$w\left(\frac{\tilde{\lambda}(q_2,q_1)}{\tilde{\lambda}(q_2,q_1)+\left(1-\tilde{\lambda}(q_2,q_1)\right)(1-a^*)}\right)=\delta V\left(\frac{q_2}{a^*},q_1\right). \tag{29}$$

Because $q_2 < \psi(q_1)$, and $V\left(\frac{q_2}{a^*}, q_1\right)$ is strictly decreasing for $a^* > \frac{q_2}{q_1}$, there exists a unique solution in $\left(\frac{q_2}{q_1}, 1\right)$.

For $q_1 \in I$, define $\tilde{\lambda}(q_2, q_1) := \frac{\lambda}{\lambda + (1-\lambda)\beta(q_1, q_2)}$. Define $\tilde{a}^*(q_2, q_1)$ to be the smallest solution to (29). Because $\delta V(r, q_1)$ is continuous in r on $[0,1]$, and $a^*(q_1, q_2) > q_1$,

$$w\left(\frac{\lambda}{\lambda + (1-\lambda)\beta(q_1, q_2)(1 - a^*(q_1, q_2))}\right) < \delta V(1).$$

Thus, strict inequality holds for $a^* = a^*(q_1, q_2) > q_1$ and for $a^* = q_2$ and thus a solution exists in $(q_2, a^*(q_1, q_2))$.

Define

$$\underline{a}^1(q_2, q_1) := \inf\left\{a \geq q_2 : V\left(\frac{q_2}{a'}, q_1\right) \geq V(q_2, q_1), \text{ for all } a' \leq a\right\}$$

and

$$\underline{a}^2(q_2, q_1) := \inf\left\{a \geq q_2 : V^h\left(\frac{q_2}{a'}, q_1\right) \geq V^h(q_2, q_1) \text{ for all } a' \leq a\right\}$$

Because $V(1, q_1) = V(1) > V(q_2, q_1)$ and $V^h(1, q_1) = V^h(1) > V^h(q_2, q_1)$ both functions are well-defined.

Define $a^*(q_2, q_1) = \min\left\{\tilde{a}^*(q_2, q_1), \underline{a}^1(q_2, q_1), \underline{a}^2(q_2, q_1)\right\}$. Define $\lambda(q_2, q_1)$ to be $\tilde{\lambda}(q_2, q_1)$ if $a^*(q_2, q_1) = \tilde{a}^*(q_2, q_1)$, and define $\lambda(q_2, q_1)$ to be such that (29) holds for $a^* = a^*(q_2, q_1)$ if $a^*(q_2, q_1) \neq \tilde{a}^*(q_2, q_1)$. ■

Let $V^h(q)$ denote an honest agency's payoff when the reputation profile is q.

Lemma 41 $\frac{\partial V^h(q_1, q_2)}{\partial q_1} > 0$ *for all* $q_1 > q_2$.

Proof. $V^h(1) = \frac{\lambda w(1)}{1-\delta} < V(1)$ and

$$V^h(q_1, q_2) = \frac{\lambda(\phi^g(q_1, q_2)) + (1-\lambda)\delta V^h(1)}{1 - \lambda\delta}$$

for all $q_1 > \rho_1^M$ because an honest agency does not give a good rating when the firm is bad. Then $V^h(q_1, q_2)$ is strictly increasing in q_1 for $q_1 > \rho_1^M$. And

$$\begin{aligned}
\frac{V^h(q_1, q_2)}{V(q_1, q_2)} &= \frac{\lambda\phi^g(q_1, q_2) + (1-\lambda)\delta V^h(1)}{\lambda\phi^g(q_1, q_2) + (1-\lambda)\delta V(1)} \\
&> \frac{V^h(1)}{V(1)} \\
&= \frac{\lambda(1 - \lambda\delta)}{1 - \delta} \\
&> \frac{\lambda\delta}{1 - \delta}.
\end{aligned}$$

For $(q_1, q_2) \in C \backslash A$,

$$V^h(q_1, q_2) = \frac{\lambda\left(\delta V\left(\chi^B(q_1, q_2)\right)\right) + (1-\lambda)\,\delta V^h\left(\chi^B(q_1, q_2)\right)}{1 - \lambda\delta}$$

and

$$\frac{V^h(q_1, q_2)}{V(q_1, q_2)} = \frac{\lambda\phi^g(q_1, q_2) + (1-\lambda)\,\delta V^h\left(\chi^B(q_1, q_2)\right)}{\lambda\phi^g(q_1, q_2) + (1-\lambda)\,\delta V\left(\chi^B(q_1, q_2)\right)}.$$

Because $\chi_1^B(q_1, q_2)$ increases with q_1, by induction, $V^h(q_1, q_2)$ increases with q_1, and

$$\frac{V^h(q_1, q_2)}{V(q_1, q_2)} > \frac{V^h\left(\chi^B(q_1, q_2)\right)}{V\left(\chi^B(q_1, q_2)\right)} > \frac{\lambda\delta}{1-\delta}.$$

For $(q_1, q_2) \in A$,

$$V^h(q_1, q_2) = \frac{\lambda\left(\delta V\left(\chi^B(q_1, q_2)\right)\right) + (1-\lambda)\,\delta\beta^*(q_1, q_2)\,V^h\left(\chi^B(q_1, q_2)\right)}{1 - \delta + (1-\lambda)\,\delta\beta^*(q_1, q_2)}.$$

Thus,

$$\begin{aligned}
& \frac{\partial V^h(q_1, q_2)}{\partial q_1} \\
= \quad & \frac{(1-\lambda)\,\delta\left[(1-\delta)\,V^h\left(\chi^B(q_1, q_2)\right) - \lambda\delta V\left(\chi^B(q_1, q_2)\right)\right]}{\left(1 - \delta + (1-\lambda)\,\delta\beta^*\right)^2}\frac{\partial\beta^*(q_1, q_2)}{\partial q_1} \\
> \quad & 0
\end{aligned}$$

because $\chi^B(q_1, q_2) \in C \backslash A$. ■