Game Theory Spring 2016
Problem Set 3: Mixed strategies

This problem set is due before class starts on Feb 29.

1. Exercise 3 in Chap 11 of the textbook. Consider a lobbying game similar to the one introduced in Chap11 of the textbook but with payoffs represented as follows

		player Y	
		L	N
player X	L	$-5, -5$	$x - 15, 0$
	N	$0, 15$	$10, 10$

Assume that $x > 25$.

   (a) Find the pure strategy Nash equilibria of this game (if it has any).

   (b) Compute the mixed strategy Nash equilibrium of this game.

   (c) Given the mixed-strategy equilibrium computed in Q1b, what is the probability that the government makes a decision favoring firm X? (It is the probability that (L, N) occurs.)

   (d) As x rises, does the probability that the government makes a decision favoring firm X rise or fall? Is this good from an economic standpoint?

2. **Private Voluntary Contribution to a Public Good.** Consider an elderly person who needs to cross the street but is unable to do so alone. There are n bystanders, who simultaneously choose whether or not to help the elderly lady cross. One person is sufficient to help the person cross the street. If the elderly person crosses the street successfully, each bystander gets utility w because he/she cares about the welfare of the elderly person. However, helping the elderly person across the street means interrupting their current activity which has an opportunity cost c independent of whether other agents are already assisting or not. Assume that $w > c$.

   (a) Consider first the case of two bystanders. Describe the normal form of this game. Write down the payoff matrix. What are the pure strategy Nash equilibria? Find the symmetric mixed strategy Nash equilibrium. That is, find a mixed strategy Nash equilibrium in which each bystander uses the same mixed strategy.

   (b) Consider now the case of three bystanders. Find the symmetric mixed strategy Nash equilibrium. How does the probability a bystander helps in the symmetric NE change when the number of bystanders increases from two to three? What is the intuition?

   (c) Consider now n bystanders and find the symmetric mixed strategy equilibrium. How does the likelihood by which an individual assists changes as n increases? How does the probability that the elderly person receives assistance changes as n increases?

3. Question 11 in Chap 11 of the textbook. The famous British spy 001 has to choose one of four routes, a,b,c, or d (listed in order of speed in good conditions) to ski down a mountain. Fast routes are more likely to be struck by an avalanche. At the same time, the notorious rival spy 002 has to choose whether to use (y) or not to use (x) his valuable explosive device to cause an avalanche. The payoffs of this game are represented here.

		Spy 002	
		x	y
Spy 001	a	$12, 0$	$0, 6$
	b	$11, 1$	$1, 5$
	c	$10, 2$	$4, 2$
	d	$9, 3$	$6, 0$

(a) Let $p_1(x)$ denote the probability that 001 believes 002 chooses x. Explain what 001 should do if $p_1(x) > \frac{2}{3}$, if $p_1(x) < \frac{2}{3}$ and if $p_1(x) = \frac{2}{3}$.

(b) Suppose you are Mr. Queue, the smart technical advisor to British military intelligence. Are there any routes you would advise 001 definitely NOT to take? Explain your answer.

(c) Find a Nash equilibrium in which one player plays a pure strategy s_i and the other player plays a mixed strategy σ_j. Find a different mixed strategy equilibrium in which this same pure strategy s_i is assigned zero probability. Are there any other equilibria?

4. Suppose that player 1's motorbike is not working properly. He does not know whether it needs an easy repair (say, a tuneup) or a major overhaul (say, an engine replacement). The probability that it needs an engine replacement is ρ. At his local mechanic, he finds that an engine replacement costs E, while a tuneup costs T ($E > T$). He knows that the

mechanic, player 2, gets the same profit, π, if she charges him for an engine replacement and indeed replaces the engine, or if she charges him for a tuneup and indeed just performs a tuneup. But she can make more profit, $\Pi > \pi$, if she charges him for an engine replacement but in fact (secretly) just does a tuneup. If it only needed a tuneup anyway, then she will get away with this, but she knows she will get sent to jail if she only does a tuneup when it needed an engine replacement. The expert is very good at her job, so she knows which is needed.

(a) Explain why player 1 should always believe player 2 when she says it just needs a tuneup, but why he might be skeptical if she says it needs a engine replacement.

Player 1 can reject player 2's advice and get a second opinion from another mechanic that never lies. Assume if he does this, however, he must accept the second mechanic's advice and accept new repair costs, $E' > E$ or $T' > T$. Since player 1 should always believe player 2 when player 2 recommends a tuneup, player 1 has two strategies: he can always accept player 2's advice, or he can reject whenever player 2 recommends an engine replacement. Player 2 can choose to give honest advice, or he can be dishonest and always recommends an engine replacement. If player 2 chooses dishonesty, then player 1 pays E if he always accepts player 2's advice, and he brings it to the second mechanic if he rejects if told "engine replacement". Here then is the game between player 1 (row) and player 2 (column).

		player 2 Honesty	Dishonesty
player 1	Always Accept Advice	$-\rho E-(1-\rho)\,T,\pi$	$-E,\rho\pi+(1-\rho)\,\Pi$
	Reject if told "engine replacement"	$-\rho E'-(1-\rho)\,T,(1-\rho)\,\pi$	$-\rho E'-(1-\rho)\,T',0$

(b) Assume that $E > \rho E' + (1-\rho)\,T'$. Explain why there is no pure strategy Nash equilibrium. Given an intuition for this condition.

(c) Find the (unique) mixed-strategy Nash equilibrium; that is, find the equilibrium randomizations in terms of the parameters.

(d) As we increase the cost of an engine replacement at the first mechanic (E) (holding all the other parameters fixed), what happens to the equilibrium probability that the expert chooses the 'honest' strategy? What happens to the equilibrium probability that player 1 chooses the strategy 'reject if told engine replacement'? Give some intuition.

(e) As we increase the profit from lying (holding all the other parameters xed), what happens to the equilibrium probability that the expert chooses the 'honest' strategy? What happens to the equilibrium probability that player 1 chooses the strategy 'reject if told engine replacement'? Give some intuition.